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Implementation of the Green-Ampt Infiltration Model: Comparative between Different Numerical Solutions

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ABSTRACT. The phenomena of infiltration and the percolation of water in the soil are of fundamental importance for the evaluation of runoff, groundwater recharge, evapotranspiration, soil erosion and transport of chemical substances in surface and groundwater. Within this context, the quantitative determination of the infiltration values is extremely important for the different areas of knowledge, in order to evaluate, mainly the surface runoff. Several types of changes in vegetation cover and topography result in significant changes in the infiltration process, making it necessary to use mathematical models to assess the consequences of these changes. Thus, this paper aims to implement the Green-Ampt model using two numerical methods - Newton-Raphson method and W-Lambert function - to determine soil permeability parameters - K and matric potential multiplied by the difference between initial and of saturation - $\psi\Delta\theta$ comparing them to the real data obtained in simulations using an automatic rainfall simulator from the Federal University of Goiás - UFG. The Green-Ampt model adjusted well to the data measured from the rain simulator, with a determination coefficient of 0.978 for the Newton-Raphson method and 0.984 for the W-Lambert function.

Keywords: rainfall simulator, Newton-Raphson method, W-Lambert function.

1 INTRODUCTION

Infiltration can be defined as a complex physical process characteristic of the soil through which water passes from the surface to its interior, connecting the surface flow with groundwater [7, 13, 18]. Infiltration is a key component in the implementation of rain-flow models [17], since this process is the main mechanism that affects the generation of runoff [36].

The infiltration values can be obtained in three ways: in situ tests, physical and numerical models. In situ tests are laborious, whether from the point of view of financial costs, equipment, test

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time and favorable climatic conditions. In contrast, physical and numerical models have gained strength in infiltration studies, either separately or together [14, 18, 20, 21, 33].

In the physical models, we can highlight the rain simulators (RS) [1, 19, 21, 33], which have as a great advantage the possibility of have controlled conditions of precipitation and physical characteristics of the soil. In addition to allowing the prediction of physical precipitation values, RS can subsidize input data for numerical models, as well as its modeling and calibration [24, 29, 31, 36].

In assessing the infiltration phenomenon, the objective is to determine the accumulated infiltration values (F) and soil infiltration rate (f). The infiltration rate is influenced by the condition of the soil surface, the type of cover and the physical, chemical and biological properties of the soil, including porosity, hydraulic conductivity and moisture content. In numerical models, the Green-Ampt equation is widely used [3, 9, 18, 22, 39] for presenting a simple structure and a clear physical concept, since it considers infiltration as a homogeneous soil profile [3, 9], and Darcy's Law can be applied.

The Green-Ampt equation for determining the accumulated infiltration is defined as [13, 16]:

$$F = Kt + \psi\Delta\theta ln(1 + \frac{F}{\psi\Delta\theta}) \tag{1.1}$$

where F is the accumulated infiltration $[L]$ at time t $[T]$, K $[LT^{-1}]$ is the hydraulic conductivity, $\psi\Delta\theta$ $[L]$ is the matrix potential multiplied by the difference in initial and saturation moisture.

The determination of the coefficients that make up Equation 1.1 is not simple and intuitive, requiring laboratory tests and numerical methods to adjust the curve to the experimental points. The difficulty in measuring these parameters is portrayed by Shao & Baumgartl [30], who proposed a set of equations, based on physical and hydraulic properties of the soil, as well as topography and vegetation of the study site, to determine the coefficient of four models infiltration, one of which is Green-Ampt.

Thus, the infiltration rate can be determined by the Equation 1.2 [13], from the accumulated infiltration value determined in Equation 1.1.

$$f = \begin{cases} i, \text{ if } i \leq f \\ K\left(1 + \frac{\psi\Delta\theta}{F}\right), \text{ if } i > f \end{cases} \tag{1.2}$$

where f is the infiltration rate for time t $[LT^{-1}]$ and i is the intensity of precipitation $[L]$.

Equation 1.1 is characterized by having an implicit solution, requiring numerical or explicit approaches to solve the problem [3, 18, 36]. Ali et al. [3] presented the main existing explicit approximations for the Green-Ampt equation, in addition to ranking the performance of each model evaluated. Through numerical methods, Enciso-Medina et al. [11] and Chowdary et al. [7] used the methods of Runge-Kutta and Newton-Raphson to solve the problem. The Newton-Raphson method stands out for its practicality in the development of the model and its application. Another numerical method used to solve the problem is to express Equation 1.1 in terms of the

W-Lambert function [5], solving the equation by approximation as proposed by Barry et. al. [5], Parlange et al. [25] and van den Putte et al. [39].

In view of all the dynamics that involve the use and application of the Green-Ampt equation for infiltration study, this paper aims to present the results of the implementation of its modeling using the Newton-Raphson methods and the W-Lambert function, implemented in MATLAB and Mathematica software, respectively, from the experimental data obtained in RS built by Sousa Júnior et al. [33]. The results predicted by the three methods were analyzed with the experimental values obtained by the RS and error metrics were used, namely relative percentage error (RPE), Willmott's agreement index (d) and the determination coefficient (R^2).

This paper aims to contribute to numerical and computational modeling studies in the areas of Geotechnics, Hydrology and Hydraulics, contemplating two widely used methods for solving the Green-Ampt equation (i.e., Newton-Raphson method and W-Lambert function) and using experimental data from rain simulators, which has become recurrent in the three large areas mentioned, either due to their cost or practicality. The differential of this paper is to obtain a comparison with advantages and disadvantages between the two methods used, supporting decision making for those who wish to implement the Green-Ampt equation in infiltration studies.

2 MATERIALS AND METHODS

In this section, the use of the rain simulator (RS) to obtain experimental data, calibration and mathematical development of the Newton-Raphson method and the W-Lambert function, applied to the Green-Ampt equation, will be discussed.

2.1 Rain simulator (RS) and physical experimental data (*in situ*) of infiltrability

To obtain the values of infiltration in the field, the RS developed by Sousa Júnior et al. [33], at the Federal University of Goiás (UFG), whose main characteristics are practicality and ease of handling and use, the ability to produce simulated rainfall with drop size values, terminal speed and kinetic energy similar to natural rains, capacity to generate a wide range of intensities and to produce a rain with uniform distribution over the experimental area (Figure 1).

The RS used is capable of simulating rain events with drops of 2.12 mm median diameter (D_{50}) and kinetic energy of 22.53 J mm^{-1} m^{-2}, which represents 90.1% of the kinetic energy produced in rains natural. The spatial distribution of the simulated rain, expressed by the Christiansen's Uniformity Coefficient (CUC) ranges from 68.6 to 90.3%. RS provides precipitations with intensities ranging from 40 to 182 mm h^{-1}. This interval covers intensities with 1 to 10 years of return and durations below 60 minutes, according to the intensity-duration-frequency curves (IDF) for the city of Goiânia, state of Goiás, Brazil [8]. In this paper, a simulated continuous rain with an intensity of 180 mm h^{-1} was used.

The built RS model consists of an "A" structure, made of 38 mm diameter iron pipe, with a total height of 3.0 m. Two Fulljet 1/2SSHH40 sprinklers, Spraying Systems Co. (USA), spaced 1.06

Figure 1: Rain simulator (RS) developed by Sousa Júnior et al. [33] and used to obtain experimental data.

m apart, were installed in a 12.7 mm PVC pipe, at a height of 2.80 m. This type of spray nozzle produces a spray pattern in the shape of a full cone, with uniform water distribution over the area for a wide range of flow rates and service pressures.

In order to obtain the experimental data of soil infiltration, a space of 1.0 m^2 was selected for testing and the runoff was measured at the outlet using a linigraph. The time taken from the water to the point of exit of the experimental plot was disregarded because it is a small value, without interfering with the final infiltration value. With the accumulated infiltration values, the K and $\psi\Delta\theta$ values were determined through a nonlinear adjustment of Equation 1.1, using the least squares method.

The least squares method consists of obtaining a minimum, close to 0, for the sum of the squares of the difference between a known value and an estimated value. Thus, an initial estimate must be arbitrated for the variables K and $\psi\Delta\theta$ and use a non-linear optimization method to solve the problem. In this paper, the least squares problem solving functions made available by MATLAB and Mathematica were used, with MATLAB using the trust region reflective algorithm and Mathematica using the Gaussian elimination method to solve the least squares problem.

2.2 Newton-Raphson method

The Newton-Raphson method (NRM) was developed to obtain roots of a function through an interactive sequence [34], represented by:

$$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}, n \in \mathbb{N} \tag{2.1}$$

where n is the step of the algorithm, x is the independent variable and $g(x)$ the function that aims to obtain the root, i.e., $g(x) = 0$.

For the application of the NRM to solve the water infiltration equation proposed by Green-Ampt [16], the variable F was considered to be independent on the function $g(F)$ (Equation 2.2), aiming to find the value of F for $g(F) = 0$. The first derivative of $g(F)$ is presented in Equation 2.3. The elements K, t, ψ, and $\Delta\theta$ are constant for each time t in the function $g(F)$. However, as presented by Shao & Baumgartl [30], obtaining these parameters is not easy, and should use optimization methods, empirical relationships or rigorous laboratory tests. The parameters K, $\psi\Delta\theta$ were obtained using the least squares method, through the trust region reflective algorithm. Such functions are already implemented in the MATLAB matrix, being used for this case.

$$g(F) = F - Kt - \psi\Delta\theta ln(1 + \frac{F}{\psi\Delta\theta}), \psi\Delta\theta \neq 0 \tag{2.2}$$

$$g'(F) = 1 - \frac{\psi\Delta\theta}{\psi\Delta\theta + F}, F \neq 0 \tag{2.3}$$

where $g'(F)$ is the first derivative of the $g(F)$ function.

The value of F for each time t is determined by the interactive process presented in Equation 2.1. As an algorithm stop criterion, the absolute error less than or equal to 0.0001 (i.e., module of the difference between the F_n and F_{n+1}). Once the value of F is determined, it is applied in Equation 1.2 and then the value of f is obtained for time t.

NRM is characterized by being a method of easy implementation, being able to be used in the most varied programs and programming languages, being implemented in the MATLAB software, version 2016, for this paper.

2.3 W-Lambert function

The W-Lambert function, in its generalized mode, is presented in Equation 2.4, and does not have a defined inverse function. Thus, the W-Lambert function proposes an inverse, presented in Equation 2.5.

$$y = xe^x \tag{2.4}$$

$$x = W(y) \tag{2.5}$$

where x is the independent variable, y is the dependent variable and $W(y)$ is the W-Lambert function.

For applicability of the W-Lambert function to the Green-Ampt equation [16], some mathematical operations are necessary. First, Equation 1.1 must be rewritten according to Equation 2.6.

$$I = T + ln(1 + I) \tag{2.6}$$

where $I = F/\psi\Delta\theta$, and $T = Kt/\psi\Delta\theta$

From Equation 2.5, it is possible to transform it into the type shown in Equation 2.4 reaching the expression presented in Equation 2.7.

$$-e^{-(1+T)} = -(1+I)e^{-(1+I)} \tag{2.7}$$

where $-e^{-(1+T)}$ would play the role of y and $-(1+I)$ would play the role of x, in Equation 2.4. Thus, the W-Lambert function can be written by Equation 2.8.

$$-(1+I) = W(-e^{-(1+T)}) \tag{2.8}$$

The W-Lambert function does not have an explicit solution, but numerical approximations can be determined to solve the problem. Parlange et al. [25] performed a reanalysis of the model by Barry et al. [5], which uses the W-Lambert function to approximate the Green-Ampt equation, and compared it with the infiltration model proposed by Talsma and Parlange [38], which also uses the W-Lambert function to approximate a solution, evaluating new limits for these functions, seeking to improve the performance of solutions.

Parlange et al. [26] proposed a unification of the Green-Ampt model [16] and Talsma and Parlange [38], developing a general formulation based on the W-Lambert function [25,36]. Swamee et al. [36] sought an explicit formulation based on the Green-Ampt [16] and Talsma and Parlange [38] models, seeking better accuracy. Swamee et al. [37] also sought a new solution to the equation of Parlange et al. [26], highlighting the use of data from rain simulators to evaluate the results obtained for the new proposed formulation.

The constants K and $\psi\Delta\theta$ were obtained by the least squares adjustment function provided by Mathematica, which uses Gaussian elimination to solve the problem.

2.4 Evaluation of numerical models

In this work, three methods were used to evaluate the results obtained by means of the numerical solution and the explicit approximation, comparing them with the actual infiltration data measured in the field, being the Relative Percentage Error (RPE), Willmott's agreement index (d) and the coefficient of determination (R^2) to assess the best fit.

The Relative Percent Error, also called Relative Error, is a simple metric used to evaluate the error between values closest to reality, in this case the values measured in the field, with the values obtained by the numerical model [3, 4, 6, 27, 32]. The RPE varies between 0 and 100% and its tolerance usually varies according to the characteristics of the data used. The RPE value can be determined by the Equation 2.9

$$\varepsilon = \frac{|y_{num} - y_{exp}|}{y_{exp}} \tag{2.9}$$

where ε is the Relative Error, y_{num} is the numerical value for the problem variable and y_{exp} is the experimental value for the problem variable.

Barry et al. [6] sought some approximations for the Green-Ampt equation, in the form of W-Lambert, and for such approximations he found a relative error ranging from 0.00005% to 0.48%. Ali et al. [3] also analyzed some approximations for the Green-Ampt equation and found a relative percentage error ranging between 0.0000001% and 12.38%. Ali & Islam [2] used a double adjustment technique on the experimental values to obtain an explicit solution of the Green-Ampt equation and found maximum values of relative percentage error ranging from 0.07% to 0.13%.

The second metric for evaluating the results obtained numerically was the determination coefficient (R^2), presented in Equation 2.10. This metric varies between 0 and 100% and determines how much a variance of a variable is explained by a certain model, being directly related to the correlation coefficient [23]. The closer to 100%, the better adjusted the model is.

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_{i,exp} - y_{i,num})^2}{\sum_{i=1}^{n} (y_{i,exp} - \bar{y}_{exp})^2} \tag{2.10}$$

where R^2 is the determination coefficient, n is the amount of data, $y_{i,exp}$ is the ith experimental value for the problem variable, $\bar{y}_{exp}$ is the average of the experimental values and $y_{i,num}$ is the ith numerical value for the problem variable.

Shao & Baumgartl [30] used this coefficient for calibration and performance of the prediction models of the parameters that make up the Green-Ampt equation, finding values that vary between 21.5% and 69.8%. Suryoputro et al. [35] evaluated five infiltration models, including the Green-Ampt model, for mineral soils in tropical regions with different types of cover and uses. For the Green-Ampt model, the authors found R^2 values ranging between 72% and 95%.

The last metric used to evaluate the results was the Willmott agreement index (d), presented in Equation 2.11 [40]. This metric is commonly used in hydrological models, varying between 0 and 1 [10, 12, 28, 41]. Duan et al. [10] used this metric to evaluate infiltration models from field values for soils with vegetation, obtaining values greater than 0.95.

$$d = \frac{\sum_{i=1}^{n} (y_{i,exp} - y_{i,num})^2}{\sum_{i=1}^{n} (|y_{i,num} - \bar{y}_{exp}| + |y_{i,exp} - \bar{y}_{exp}|)^2} \tag{2.11}$$

where d is the Willmott agreement index, n is the amount of data, $y_{i,exp}$ is the ith experimental value for the problem variable, $\bar{y}_{exp}$ is the average of the experimental values and $y_{i,num}$ is the ith numerical value for the problem variable.

3 RESULTS

Table 1 shows the infiltration values measured in the field using the RS developed by Sousa Júnior et al. [33] and the main parameters obtained in each proposed resolution method, using the algorithms implemented in MATLAB and Mathematica software. The use of different software was due to the best tools that each program can provide for their respective methods, facilitating implementation.

Table 1: Results of the Green-Ampt infiltration model for the Newton-Raphson and W-Lambert method for a simulated rain event of 180 mm h^{-1}.

t (min)	**P (mm)**	**MATLAB (NRM)**		**Mathematica (W-Lambert function)**	
		***f* (mm h^{-1})**	***F* (mm)**	***f* (mm h^{-1})**	***F* (mm)**
0	0	180.0	3.00	180.0	3.00
1	3	180.0	3.00	180.0	3.00
2	6	173.3	5.94	158.1	5.11
3	9	142.1	8.64	144.2	7.96
4	12	133.5	11.04	136.1	10.48
5	15	129.8	13.29	130.7	15.02
6	18	124.3	15.44	126.9	17.17
7	21	124.3	17.52	124.0	19.26
8	24	120.0	19.55	121.7	21.31
9	27	119.4	21.54	119.8	23.32
10	30	118.2	23.5	118.2	25.3
11	33	117.0	25.44	116.9	27.26
12	36	115.1	27.35	115.8	29.21
13	39	114.5	29.24	114.8	31.12
14	42	113.9	31.11	113.9	33.03
		K = 97.2 mm h^{-1}	**$\psi\Delta\theta$ = 4.62 mm**	K = **99.9 mm h^{-1}**	$\psi\Delta\theta$ = 4.64 mm

In Table 1, it can be seen that both the coefficients of the Green-Ampt equation, as well as the infiltration rate and accumulated infiltration values, remained close for the two numerical methods used. For the saturated hydraulic conductivity value, a value slightly higher than the maximum value obtained by Shao & Baumgartl [30] was obtained. It is worth noting that the K values have great variability, either by the type of soil analyzed or by the test method used. Gitirana and Fredlund [15] showed that $ln(K)$ has a coefficient of variation of 20.6%. Calculating the relative percentage error of the K values with the maximum obtained by Shao & Baumgartl

[30], an average error for the two methods of 2.20% is obtained, considered low in view of the high value of the variation coefficient. Suryoputro et al. [35] obtained a wide variation in the K value for the Green-Ampt equation, ranging from 0.06 to 429.6 mm h^{-1}, an interval that covers the K values obtained in this paper.

The $\psi\Delta\theta$ values obtained in this paper were shown to be within the range obtained by Shao & Baumgartl [30], but it is worth noting that the values of matric potential, also called matrix suction, obtained by these authors varied considerably, with an interval of 4705.96 mm, between the maximum and minimum value. In fact, the matrix suction values have a lot of variability and sensitivity, and are quite costly to determine in the field. Gitirana Jr. & Fredlund [15] analyzed the parameters that make up the soil-water characteristic curve (i.e., suction vs. volumetric water content), and for the suction parameters that make up the curve fitting equation, the authors obtained coefficients of greater than 5000%, showing the great variability and sensitivity of these data. Suryoputro et al. [35] obtained very varied suction values for the Green-Ampt model, ranging from 29.20 mm to 1068.50 mm, showing the great variability of this parameter.

As for the infiltration rate and accumulated infiltration data, the values were close to those obtained by Formiga et al. [13], who used data for an intensity rain of 180 mm h^{-1}. It is noteworthy that between the two studies, the first values were the same, even due to the condition presented in Equation 1.2, but the final simulation values showed a small variation. Formiga et al. [13] obtained the infiltration values through a numerical adjustment of the experimental data to the Green-Ampt equation.

In Figure 2, it shows the behavior of the infiltration rate (f) over the test time and the numerical estimates used. It is noticed that in the first two simulation times the two methods are the same as the one obtained by the physical test, being justified by the condition imposed in Equation 1.2. With 2 minutes of rain, the physical test still remains with 180 mm imposed by the condition of the Equation 1.2, however, numerical methods already have a lower infiltration rate than precipitation. It must be remembered that the infiltration process is not only controlled by the variables K and $\psi\Delta\theta$, as presented by the Green-Ampt equation, but also by the morphology and structure of the soil, the presence of organic materials and the chemical composition of the soil. In addition, the measurement of infiltration values in the field is also a difficult process, with several types of tests, which have different degrees of precision. Despite this, after the first two minutes of rain, the precipitation values are very close, and it is clear that after 9 minutes the results obtained by W-Lambert adjust to the physical values. In general, the Newton-Raphson method was found to underestimate precipitation values. The values obtained by the W-Lambert function were shown to be more continuous, which can be justified by the fact that this method rewrites the Green-Ampt equation as a form of a new Function, presented in Equation 2.8.

The values of relative percentage error (ε) are shown in Figure 3. It is noteworthy that despite Equation 2.9 using the module in the denominator, it was decided to present the sign of the values in the graph, since this sign has a meaning, being that for positive values there is an overestimation of the numerical values in relation to the experimental one, and for negative values there is the opposite process, of underestimation. Thus, it is clear that the Newton-Raphson method un-

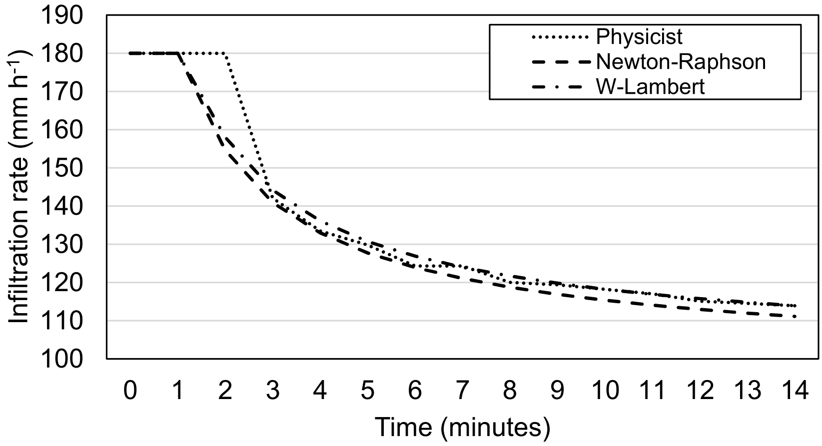

Figure 2: Comparison of the infiltration rate observed with those calculated by the Newton-Raphson and W-Lambert methods for the Green-Ampt infiltration equation.

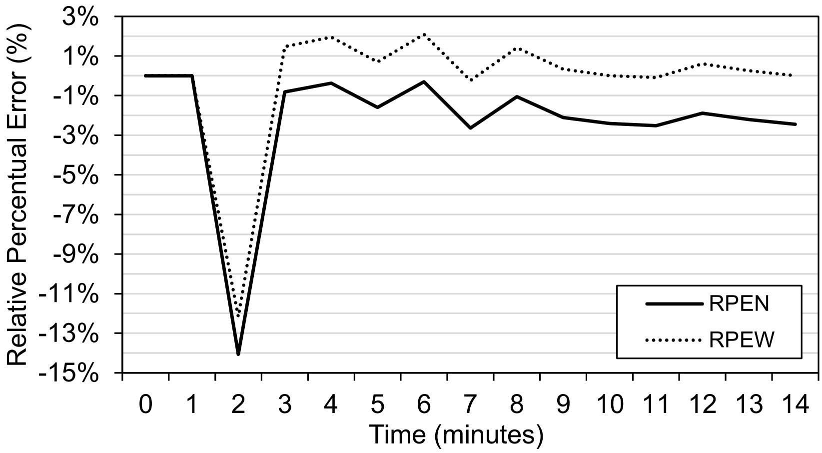

Figure 3: Relative percentage error for Newton-Raphson (EPRN) and W-Lambert (EPRW) methods.

derestimated the experimental values, while W-Lambert presented the opposite behavior. It can be seen that the W-Lambert errors remained closer to 0%, especially for the last minutes, showing the good adjustment mentioned in Figure 2. In addition, it is possible to notice that for the two methods used there was an underestimation of the values infiltration rate and accumulated rate after 2 minutes of rain, since the physical test still remained with the condition of equality between the infiltration rate and precipitation value.

In general, the two methods showed a good coefficient of determination (R^2), whereas for the Newton-Raphson method it was 90.75%, and for the W-Lambert method it was 92.91%. Formiga et al. [13], using simulated rain data for an intensity of 180 mm h^{-1}, found an R^2 of 97.0%.

Suryoputro et al. [35] obtained R^2 values of 76.0% for the Green-Ampt model. Both results show that NRM and W-Lambert presented good values of coefficient of determination, being among values found in the literature.

As for the Willmott conformity index values (d), values of 0.978 and 0.984 were obtained, for the Newton and W-Lambert method, respectively. These values show that the results obtained showed to be quite adjusted as to the experimental values, showing the efficiency of the numerical methods for solving the Green-Ampt equation.

About the algorithms used to obtain the numerical solution, the need for an initial value, with input by the user, for the Newton-Raphson method, in order to find a solution for each instant of time, stands out. The first initial value, for the time period of 0 minutes, used in the algorithm was 0.1 mm. This value was chosen because in that instant of time the rain starts, having a very small value of accumulated infiltration rate. For the other times, the initial value has always been the value obtained in the previous time. Since the accumulated infiltration rate tends to increase over time, using the value obtained in the previous time becomes an artifice for quickly obtaining the numerical solution. Regarding the computational costs of such methods, both proved to be quite fast, with no need for more powerful computers or systems for their execution.

4 CONCLUSIONS

The numerical methods used in this paper proved to be very promising and useful in determining the infiltration values by the Green-Ampt equation, showing maximum errors of 12% for the W-Lambert function and 14% for the Newton-Raphson method. The W-Lambert function proved to be little better than the Newton-Raphson method. It is noteworthy that the Newton-Raphson method is easier to apply, managing to be more practical and presenting good values compared to the physical model. However, the W-Lambert function is also promising for determining equations with explicit Green-Ampt solutions, with good mathematical use for this purpose. Both methods showed good behavior even with the great variability of the parameters that make up the Green-Ampt infiltration model.

For future work, it is suggested to use these numerical methods with prediction functions, as proposed by Shao & Baumgartl [30], of the parameters of the the Green-Ampt infiltration model (i.e., K, t, ψ, and $\Delta\theta$), showing an interesting way to determine infiltration values, either rate or accumulated, without the need for large trials, which may require cost and time.

Regarding the use of rain simulators (RS) to validate the infiltration equations used, the equipment proved to be quite applicable and indicated, as it was not necessary to wait for natural rain occurrences, being possible to simulate artificial rains at any time with characteristics very close to the real rains, in addition to allowing greater control of the variables involved and less variability.

Thus, as long as the field conditions are very well represented by the laboratory conditions, the RS can complement and subsidize information with the numerical methods, proving to be an important path for a better understanding of the phenomenon of infiltration in the field.

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