# Damping of Low-Frequency Oscillations by Supplementary Control of Power System Stabilizers

C.A.T. FEBRES<sup>1</sup>, Departamento de Engenharia de Energia e Automação Elétricas, USP, 05508-970 São Paulo, SP, Brasil.

P.B. ARAUJO<sup>2</sup>, M.A. FURINI<sup>3</sup>, Departamento de Engenharia Elétrica, FEIS, UNESP, 15385-000 Ilha Solteira, SP, Brasil.

**Abstract**. This work presents studies of the small signal stability applied to an electric power system with the consideration of the Power System Stabilizer in operation. The Power Sensitivity Model is used to represent the electric power system. Information about the stability of the electric power system when subjected to small disturbances is obtained simulating a test system.

**Keywords**. Electric Power Systems. Small-Signal Stability. Power System Stabilizers.

### 1. Introduction

Small-Signal Stability is a fundamental condition for a safe and reliable operation of Electric Power Systems, so this topic is fairly studied in the literature [2, 5, 8]. This paper presents results of small-signal stability studies of a multi-machine power system, considering the Power System Stabilizers (PSS) as additional source of damping to low-frequency electromechanical oscillations, (local modes - characterized by frequencies in the range of 0,8 Hz to 2,0 Hz). Since PSS is a local controller, a local signal, deviations of generator angular speed [9], is used as its input. The Power Sensitivity Model (PSM) is used to represent the electric power system [4]. Participation factors and residues will show which generators should be equipped with PSS [1, 6, 7, 12]. The phase compensation method will be used to adjust the PSS parameters. Simulations of the South-Brazilian Reduced [10] system will give assessments and conclusions about stability of power systems.

# 2. Power Sensitivity Model

Power Sensitivity Model is a linear analysis tool for the electric power systems. The fundamental concept is the application of active and reactive power balance

<sup>&</sup>lt;sup>1</sup>carfeb@gmail.com

 $<sup>^{2} {\</sup>it percival@dee.feis.unesp.br}$ 

<sup>&</sup>lt;sup>3</sup>mafurini@aluno.feis.unesp.br

for all the buses (nodes) of the system as shown by equations (2.1), where  $\Omega_k$  is the neighbour buses set of a generic bus k [4].

$$\Delta P_{Gk} - \Delta P_{Lk} - \sum_{m \in \Omega_k} \Delta P_{km} = 0$$
  
$$\Delta Q_{Gk} - \Delta Q_{Lk} - \sum_{m \in \Omega_k} \Delta Q_{km} = 0$$
(2.1)

In equations (2.1) subindices G, L and km represent respectively power generated, load power and power flow through a transmission line.

The PSM multi-machine modeling adopted in this paper uses as dynamical variables for each generator the angular velocity  $(\omega)$ , the rotor angle  $(\delta)$ , the internal voltage of quadrature axis  $(E'_q)$  and the electrical field voltage  $(E_{fd})$ . The input variables of this system are defined as the input mechanical power  $(P_m)$  and the reference voltage  $(V_{ref})$  for the automatic voltage regulator of each generator. The buses are represented by the module (V) and angle  $(\theta)$  of terminal voltage.

The variables described above and the equations defining them must be linearized around an equilibrium point in a way on which they could be grouped in state variable vectors  $(\Delta x)$ , algebraic variables  $(\Delta z)$  and input variables  $(\Delta u)$ , as shown in the equations (2.2).

$$\begin{bmatrix} \Delta x \end{bmatrix} = \begin{bmatrix} \Delta \omega & \Delta \delta & \Delta E'_q & \Delta E_{fd} \end{bmatrix}$$
$$\begin{bmatrix} \Delta z \end{bmatrix} = \begin{bmatrix} \Delta V & \Delta \theta \end{bmatrix}$$
$$\begin{bmatrix} \Delta u \end{bmatrix} = \begin{bmatrix} \Delta P_m & \Delta V_{ref} \end{bmatrix}$$
(2.2)

The dynamical behavior of the system is represented on the time domain according to the equation (2.3).

$$\begin{bmatrix} \Delta \dot{x} \\ 0 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \Delta u$$
(2.3)

The electric power system representation on the frequency domain, shown on Figure 1, is obtained from Laplace Transform of linearized system equations for a generic generator k.

### 3. Modal Analysis

#### 3.1. Eigenvalues

The Small-Signal Stability of electric power systems is given by the eigenvalues of the state matrix A. Eigenvalues may be real or complex. A real eigenvalue represents a non-oscillatory mode, so if it is positive it corresponds to an aperiodic instability. However, a complex pair of eigenvalues corresponds to a particular oscillation mode. The real part of complex eigenvalues provides the damping coefficient, while the imaginary part gives the oscillation frequency. Thus, if the real part is negative, these oscillations decrease. In the case of some real and positive value, oscillations increase due to the lack of damping. The eigenvalues ( $\lambda$ ) of the state matrix are given by the non-trivial solutions of the equation (3.1).



Figure 1: Power Sensitivity Model - Block Diagram.

$$A\varphi = \lambda\varphi \tag{3.1}$$

Equation (3.2) can be written from equation (3.1), whose solutions are the eigenvalues of state matrix. Equation (3.2) is well recognized as the characteristic equation of the system.

$$\det\left(A - \lambda I\right) = 0\tag{3.2}$$

#### 3.2. Eigenvectors

For each eigenvalue  $\lambda$  of the state matrix, there are right ( $\varphi$ ) and left ( $\psi$ ) eigenvectors which satisfy the equations (3.3) and (3.4), respectively.

$$A\varphi_i = \lambda_i \varphi_i \tag{3.3}$$

$$\psi_i A = \lambda_i \psi_i \tag{3.4}$$

In the stability analysis from eigenvectors some variables are defined, which can be observed (*n* state variables) and its contribution for each unstable oscillatory mode through participation factors  $fp_{ni}$  can be obtained as follows according to the equation (3.5).

$$fp_{ni} = \varphi_{in}\psi_{ni} \tag{3.5}$$

The participation factor shows the relative participation of the state variable n in the mode i. These indices are dimensionless quantities that aid to identify possible sources of problems related to the poorly damped and/or unstable modes. From the participation factors the best generators to be equipped with PSS to provide effective degree of damping can be determined [1, 3, 11].

#### 3.3. Residues

Another way to obtain the best place to install PSS controllers is to use the residues of the open-loop transfer function of this device. The residues  $(R_{ijk})$  provide information about the observability and controllability of the PSS input-output set  $(C_j - B_k)$  in a predefined eigenvalue  $(\lambda_i)$  according to the right  $(\varphi_i)$  and left  $(\psi_i)$ eigenvectors as described in equation (3.6).

$$R_{ijk} = C_j \varphi_i \psi_i B_k \tag{3.6}$$

Therefore, the bigger the residues obtained the bigger the damping insert on the system. It is emphasized that the residues obtained from each possible installation of the PSS provide the best localization of these devices on the system [6].

### 4. Power System Stabilizer

The PSS is designed to introduce an electrical torque in phase with the rotor speed variations (damping torque). This is achieved by a supplementary stabilizing signal  $\Delta V_s$  applied to the automatic voltage regulator (AVR) of the generator as shown in Figure 2. This Figure also exemplifies the PSS basic structure to promote phase compensation to the phase lag introduced by generator, excitation system and transmission system [1, 9].



Figure 2: PSS Basic Structure and Supplementary Signal to AVR.

Basically, this controller is composed of a static gain  $K_{pss}$  which is adjusted to obtain the desired damping for unstable or poorly damped modes; a washout block is defined by the time constant  $T_w$  (in the range of 1 to 20 seconds), it works as a filter for low-frequencies (0,8 to 2,0 Hz); the time constants  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  define two blocks lead-lag of the input signal [8].

226

#### 4.1. Inclusion of Power System Stabilizers in the PSM

The inclusion of PSS in the PSM is formulated by the equations (4.1) to (4.4), which are obtained from Figure 2.

$$\Delta \dot{V}_1 = K_{PSS} \Delta \dot{\omega} - \frac{1}{T_w} \Delta V_1 \tag{4.1}$$

$$\Delta \dot{V}_{2} = \frac{T_{1}}{T_{2}} \Delta \dot{V}_{1} + \frac{1}{T_{2}} \Delta V_{1} - \frac{1}{T_{2}} \Delta V_{2}$$
(4.2)

$$\Delta \dot{V}_{s} = \frac{T_{3}}{T_{4}} \Delta \dot{V}_{2} + \frac{1}{T_{4}} \Delta V_{2} - \frac{1}{T_{4}} \Delta V_{s}$$
(4.3)

$$\Delta \dot{E}_{fd} = -\frac{1}{T_r} \Delta E_{fd} + \frac{K_r}{T_r} \Delta V_{ref} - \frac{K_r}{T_r} \Delta V_k + \frac{K_r}{T_r} \Delta V_s \tag{4.4}$$

Analyzing the equations (4.1) to (4.4), it is observed that the action of PSS whose input signal are the variations of the angular speed, introduce three new state variables, namely  $\Delta V_1$ ,  $\Delta V_2$  and  $\Delta V_s$ .

#### 4.2. Design of PSS Parameters

Once the residues are obtained, this index gives the procedure to design and obtain the PSS parameters.

To design the controller, it is necessary to calculate the time constants  $T_1 = T_3$ ,  $T_2 = T_4$  and the gain  $K_{pss}$  in order to introduce the necessary phase compensation to the displacement of the eigenvalue of interests. An eigenvalue  $(\lambda_i)$  can be displaced to the left semi-plane including PSS, in such way that the real part becomes more negative, increasing the damping of the oscillatory mode. Equation (4.5) shows the relation between the displacement of the eigenvalue and the correspondent residue [6].

$$\Delta \lambda_i = R_{ijk} PSS\left(\lambda_i\right) = R_{ijk} \left(K_{pss} H\left(\lambda_i\right)\right) \tag{4.5}$$

Thus, consider that the angle to be compensated by the controller is  $\beta$ , and  $\omega_i$  is the frequency in rad/s of the electromechanical mode of interests, and  $\lambda_{ides}$  the desired position of the eigenvalue, then the equation (4.6) provides the procedure to obtain the parameters of the controller.

$$\beta = 180^{\circ} - \arg(R_{ijk}); \quad \alpha = \frac{1 - \sin\left(\frac{\beta}{2}\right)}{1 + \sin\left(\frac{\beta}{2}\right)}; \quad K_{pss} = \left|\frac{\lambda_{ides} - \lambda_i}{R_{ijk}H(\lambda_i)}\right|. \quad (4.6)$$

$$T_2 = \frac{1}{\omega_i \sqrt{\alpha}}; \quad T_1 = \alpha T_2;$$

## 5. Simulation and Results

The South-Brazilian Reduced system, whose diagram is shown in Figure 3, has 10 generators, 45 buses and 73 transmission lines and will be used in the simulations.

Mode	Eigenvalues	ζ	$\omega_n$
1	$-0,2623 \pm 9,6782i$	0,0271	$1,\!5409$
2	$-0,1120 \pm 9.6155i$	0,0116	1,5305
3	$-0,0269 \pm 8.6784 i$	0,0031	1,3812
4	$0,1715 \pm 8.0573 i$	-0,0213	1,2826
5	$0,0838 {\pm} 6.1661 \mathrm{i}$	-0,0136	0,9815
6	$-0,0125 \pm 7.6730i$	0,0016	1,2212
7	$0,1194{\pm}7.2562i$	-0,0164	$1,\!1550$
8	$0,0847 \pm 6.7135i$	-0,0126	1,0686
9	$0,0322 \pm 6.9301$ i	-0,0046	1,1030

Table 1: Dominant Eigenvalues

By using PSM we calculate the eigenvalues of state matrix, whose dominant values, the damping coefficient and the natural frequency of oscillation associated with them are shown in Table 1.

Analyzing the eigenvalues listed in Table 1 it is noted that this system, for the considered operation condition, has nine oscillating modes, where five among them are unstable (modes 4, 5, 7, 8 and 9). From the values of the natural frequencies it is observed that these modes are unstable local modes of oscillation. To lead the system to stability it is proposed the inclusion of PSSs, whose parameters will be adjusted in accordance with the procedure described previously.

The participation factors related to unstable modes were calculated and shown in Figure 4 in a way to define the best generators to be equipped with PSS.

The participation factors show that the variables  $\Delta \delta_1$ ,  $\Delta \omega_1$ ,  $\Delta \delta_3$ ,  $\Delta \omega_3$ ,  $\Delta \delta_4$ ,



Figure 3: South-Brazilian Reduced System.



Figure 4: Participation Factors for Unstable Modes.

 $\Delta\omega_4$ ,  $\Delta\delta_8$  and  $\Delta\omega_8$  have bigger participation in unstable modes, which indicates that generators 1, 3, 4 and 8 are the best locations for the introduction of stabilizing signals.

Another way to determine the best location for the PSS is obtained by the analysis of residue from open-loop transfer function of each controller to be inserted in each generator. So consider Table 2, which shows the magnitude of the residues associated with the unstable modes and all possible locations of the PSS in the system.

According to Table 2 it can be conclude that the residues magnitude gives the same results as the ones presented by participation factors. From these locations, parameters of four stabilizers were calculated by the method described in section 4.2 for two desired values of damping coefficients: 0,05 and 0,01. The results are shown in Table 3.

Note in Table 3 that the time constants are the same for both settings, because the compensation is the same for both cases. In contrast to the gain values, which

Generator	Mode 4	Mode 5	Mode 7	Mode 8	Mode 9
1	0,0030332	0,0045943	$0,\!1120800$	0,0003783	0,0024259
2	0,0000080	0,0145220	0,0000734	0,0053895	0,0001067
3	0,0000818	$0,\!0344450$	0,0010043	0,0356440	0,0027326
4	0,0816110	0,0054661	0,0060700	0,0018071	0,0019654
5	0,0698140	0,0038643	0,0151350	0,0015881	0,0044152
6	0,0001159	0,0002736	0,0001891	0,0007784	0,0125260
7	0,0001647	0,0004012	0,0003044	0,0011009	0,0180030
8	0,0004242	0,0009245	0,0006669	0,0026664	0,0420800
9	0,0001471	0,0100940	0,0023453	0,0340320	0,0048138
10	0,0002569	0,0032999	0,0063526	0,0011332	0,0022884

Table 2: Residues associated to the unstable modes.

 Table 3: PSS Parameters

Generator	$K_{pss} - \zeta = 0,05$	$K_{pss} - \zeta = 0,01$	$T_w$	$T_1$	$T_2$
1	8,241	$3,\!279$	1,0000	$0,\!1895$	0,1001
3	14,784	$5,\!484$	1,0000	0,1834	0,1434
4	$15,\!852$	6,955	1,0000	$0,\!1853$	0,0831
8	13,810	3,700	1,0000	$0,\!1777$	0,1171

increase when it is required a bigger damping.

Eigenvalues, damping coefficients and natural frequencies are shown in Table 4 due to installation of four PSS with the parameters shown in Table 3 ( $\zeta = 0,01$ ). Note that the adjustments are very satisfactory, because the system has become stable with positive damping coefficients for all eigenvalues.

Table 4: Dominant Eigenvalues with PSSs -  $\zeta = 0,01$ 

Mode	Eigenvalues	ζ	$\omega_n$
4	$-0,4555\pm7,2581i$	0,0626	$1,\!1574$
5	$-0,2908\pm6,2150i$	0,0467	0,9902
7	-0,4555±7,2581i	0,0626	$1,\!1574$
8	$-0,1865\pm6,6330i$	0,0281	1,0561
9	$-0,2838 \pm 6,9983 \mathrm{i}$	0,0405	1,1147

Figure 5 shows the variations of the angular speed of the generator 3 due to a disturbance of 0.05 pu in the input mechanical torque of generator 1 (generator 1 was considered the reference for the system test).



Figure 5: Variations of Angular Speed - Generator 3.

Analyzing Figure 5 it is observed that the increase of damping coefficient creates more damping for oscillations, and the system achieve a new stable operation point more quickly.

### 6. Conclusions

This paper presented studies on the performance of the PSS controllers for the damping of low-frequency oscillations in a electric power system. The PSM was used to represent the multi-machine system. Following, the PSS controllers were introduced, using as input signal the variations of angular speed ( $\Delta \omega$ ) in order to introduce damping.

The location of stabilizers was determined by participation factors and the adjustment of its parameters was carried out in accordance with the phase compensation method. Through simulations on the South-Brazilian Reduced System it is concluded the efficiency of PSS to promote stabilization of electric power systems.

**Resumo**. Este trabalho apresenta estudos sobre a Estabilidade Dinâmica de um Sistema Elétrico de Potência com a consideração do Estabilizador do Sistema de Potência. O Modelo de Sensibilidade de Potência foi utilizado para representação do sistema. Através de simulações de um sistema teste serão obtidas informações a respeito da estabilidade do sistema quanto submetido a pequenas pertubações.

### References

- E.H. Abed, D. Lindsay, W.A. Hashlamoun, On participation factors for linear systems, *Automática*, 36 (2000), 1489-1496.
- [2] P. Anderson, A.A. Fouad, "Power Systems Control and Stability", John Wiley & Sons INC., New York, 2003.

- [3] L.C. Chen, Y.Y. Hsu, An efficient algorithm for the design of decentralized output feedback power system stabilizer, *IEEE Transactions on Power Systems*, 3, No. 3 (1988), 999-1004.
- [4] S.M. Deckmann, V.F. da Costa, A power ensitivity model for electromechanical oscillation studies, *IEEE Transactions on Power Systems*, 9, No. 2 (1994), 965-971.
- [5] F.P. DeMello, C. Concordia, Concepts of synchronous machine stability as affected by excitation control, *IEEE Transactions on Power Apparatus and Systems*, 88, No. 4 (1969), 316-329.
- [6] M.A. Furini, P.B. de Araujo, Melhora da estabilidade dinâmica de sistemas elétricos de potência multimáquinas usando o dispositivo FACTS "Thyristor Controlled Series Capacitor - TCSC", SBA Controle & Automação, 19, No. 2 (2008), 214-225.
- [7] C.H.C. Guimarães, G.N. Taranto, S. Gomes Jr., N. Martins, Projeto de estabilizadores de sistemas de potência por posicionamento parcial de pólos complexos conjugados, in "Anais do XIII Congresso Brasileiro de Automática", pp. 1-6, CBA, 2000.
- [8] P. Kundur, "Power Systems Stability and Control", Electrical Power Research Institute Series, McGraw Hill, New York, 1994.
- [9] E.V. Larsen, D.A Swann, Applying power system stabilizers, Part I: general concepts, Part II: objectives and tuning concepts, Part III: practical considerations, *IEEE Transactions on Power Apparatus and Systems*, PAS-100, No. 12 (1981), 3017-3046.
- [10] C.R. Minussi, "Controle de Segurança Dinâmica em Sistemas de Energia Elétrica", Tese de Doutorado, UFSC, Florianópolis, SC, 1990.
- [11] I.J. Pérez-Arriaga, L. Rouco, F.L. Pagola, J.L. Sancha, The role of participation factors in reduced order eigenanalysis of large power systems, in "IEEE International Symposium on Circuits and Systems", pp. 923-927, IEEE, 1988.
- [12] G.C. Verghese, I.J. Pérez-Arriaga, F.C. Schweppe, Selective modal analysis with applications to electric power systems, Part II: the dynamic stability problem, *IEEE Transactions on Power Apparatus and Systems*, PAS-101, No. 9 (1982), 3126-3134.