

An Integrated Control Model (Chemical and Biological) via Fuzzy Linear Programming for *Spodoptera frugiperda* (Lepidoptera: Noctuidae)

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ABSTRACT. This paper proposes an integrated control model that enables the pesticide use according to the prey density and the insertion of the natural enemy, the predators, by fuzzy linear programming. The fall armyworm *Spodoptera frugiperda* (Lepidoptera: Noctuidae) is one of the main resistant pests of corn, which can cause damage to the productivity up to 34%. The control is strictly chemical.

Keywords: fall armyworm, control, predators.

1 INTRODUCTION

Brazil is one of the major exporters of agricultural products, focusing on corn, one of its main crops, leading the world export in 2019, surpassing quantities previously exported [4]. In this perspective, the need for risk management that minimizes costs and maximizes productivity increases and one of the major problems is associated with the incidence of pests in plantations, given that the costs related to control are high.

Therefore, this work proposes an integrated control model via Linear Fuzzy Programming for fall armyworm *Spodoptera frugiperda* (Lepidoptera: Noctuidae), one of the main pests of corn consolidated nationally, which can generate losses of up to 34% in production.

Linear Programming (LP) is the field of knowledge of Mathematical Programming that aims to solve problems in a quantitative manner with the purpose of making the best decisions. In this context, the decision variables are continuous and present linear behavior, both in relation to the objective function and the constraints. However, in several practical situations, the constraints, or even the objective function in LP problems can not be represented by precise values. This

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kind of uncertainty can be modeled by a fuzzy number. The Fuzzy Sets Theory can model the approximate reasoning, which attempts to get closer to the human ability to make decisions, considering uncertainties and inaccuracies inherent to the data or to the processes [1, 6, 8].

In general, physical and biological phenomena are well modeled when considering intrinsic uncertainties. In particular, some features are well understood by language terms, in contrast to the traditional mathematical treatment. In this context, Fuzzy Sets Theory emerged in 1965 [17], making contributions to the approach of uncertainties inherent in language itself, and proposing a conjunctist treatment to deal with subjective and uncertain aspects and attributes [6].

Fuzzy Linear Programming is a family of optimization problems in which the optimization model parameters are not well defined, that is, the objective function and/or constraint coefficients are not exactly known and that some of the inequalities involved may also be subject to unsharp boundaries [14]. This is our case, because the precise values of the parameters are not known and approximations can be specified more comfortably than point values.

2 PRELIMINARIES

2.1 On *Spodoptera frugiperda* (Lepidoptera: Noctuidae)

The average percentage crop losses in corn caused by the caterpillar *S. frugiperda* is 15 % in the first 30 days, reaching 34% in the flowering phase [2]. Migrant moths from other areas lay their eggs on the leaves, and the newly hatched larvae begin to feed on the green tissues reducing the leaf area of the plant. As they grow, they make holes in the leaves being able to destroy completely the young plants as well can affect larger plants. The attack can occur from 10 days after seedling emergence until ear formation, the most developed larvae can consume the interior of the culm (stem), causing the death of the plant, or penetrate the base of the ear, damaging the grains. and making it susceptible to the entry of pathogens. The main control technique for fall armyworm has been based on the application of pesticides, however, there are numerous disadvantages associated with the use of this type of control, such as cases of resistance, high cost, toxicity to the environment and to workers. Thus, there is a demand for the use of other methods, including natural biological control.

The natural biological control of pests is one of the most important bases in an integrated management program, considering that it makes it possible to manage agricultural crops prioritizing a balanced interaction with the environment, especially when other control tactics are used, such as chemical with pesticides. selective to natural enemies [3]. Under those considerations, this work proposes an integrated management model in order to determine the amount of viable pesticide per area considering the prey-predator interaction of *S. frugiperda* and the scissors *Doru luteipes*.

2.2 Using Fuzzy Linear Programming

The classical linear programming problem is to find the minimum (or maximum) values of the linear function under constraints represented by linear inequalities or equations, that is,

$$\begin{aligned} & \text{maximize (or minimize)} && f(x) = c^T x \\ & \text{subject to :} && Ax \leq b \\ & && x \geq 0 \end{aligned}$$

where the function f is called an objective function and $x = (x_1, x_2, \dots, x_n)^T$ is a vector of variables. The vector $c = (c_1, c_2, \dots, c_n)$ are called cost vector. The matrix $A = [a_{ij}]$, where $i \in \mathbb{N}_m$ and $j \in \mathbb{N}_n$, is called a constraint matrix, and the vector $b = (b_1, b_2, \dots, b_m)^T$ is called a right-hand-side vector. As is well know, many practical problems can be formulated as linear programming problems [12].

However, in many practical situations, it is not reasonable to require that the constraints or objective functions in linear programming problems be specified in crisp terms. In these situations, it is desirable to use some type of fuzzy linear programming [11].

Fuzzy linear programming (FLP) is a family of optimization problems in which the optimization model parameters are not well defined, that is, the objective function and/or constraint coefficients are not exactly known and that some of the inequalities involved may also be subject to more flexible limits [14].

The optimization model associated with a linear programming problem in which only the right-hand-side numbers B_i ($i \in \mathbb{N}_m$) are fuzzy numbers is formulated as follows:

$$\text{max (ou min)} \quad \sum_{j=1}^n c_j x_j \tag{2.1}$$

$$\text{subject to :} \quad \sum_{j=1}^n a_{ij} x_j \leq B_i \quad i = 1, 2, \dots, m \tag{2.2}$$

$$x_j \geq 0 \quad j = 1, 2, \dots, n \tag{2.3}$$

where B_i ,

$$B_i = \begin{cases} 1 & , \text{se } x \leq b_i \\ \frac{b_i + p_i - x}{p_i} & , \text{se } b_i < x < b_i + p_i \\ 0 & , \text{otherwise} \end{cases} \tag{2.4}$$

in which $x \in \mathbb{R}$.

In general, fuzzy linear programming problems are first converted into equivalent crisp linear or nonlinear problems, which are then solved by standard methods. The final results of a fuzzy linear programming problem are thus real numbers, which represent a compromise in terms of the fuzzy numbers involved. Next, we determine the fuzzy set of optimal values. This is done

by calculating the lower and upper bounds of the optimal values first. The lower bound of the optimal values, z_l , is obtained by solving the FLP:

$$\max(\text{oumin}) \quad z = cx \tag{2.5}$$

$$\text{subject to :} \quad \sum_{j=1}^n a_{ij}x_j \leq b_i \quad i = 1, 2, \dots, m \tag{2.6}$$

$$x_j \geq 0 \quad j = 1, 2, \dots, n \tag{2.7}$$

and the upper bound of optimal values, z_s , is obtained by the simply substitute b_i por $b_i + p_i$:

$$\max(\text{oumin}) \quad z = cx \tag{2.8}$$

$$\text{subject to :} \quad \sum_{j=1}^n a_{ij}x_j \leq b_i + p_i \quad i = 1, 2, \dots, m \tag{2.9}$$

$$x_j \geq 0 \quad j = 1, 2, \dots, n \tag{2.10}$$

Therefore, the fuzzy FLP in (2.1) becomes a classic FLP,

$$\max(\text{oumin}) \quad \lambda \tag{2.11}$$

$$\text{subject to :} \quad \lambda(z_s - z_l) - cx \leq -z_l \tag{2.12}$$

$$\lambda p_i + \sum_{j=1}^n a_{ij}x_j \leq b_i + p_i \quad i = 1, 2, \dots, m \tag{2.13}$$

$$x_j \geq 0 \quad j = 1, 2, \dots, n \tag{2.14}$$

$$\lambda \in (0, 1] \tag{2.15}$$

In following section provides an control for a corn pest by FLP.

3 THE MATHEMATICAL MODEL

The earwig *D. luteipes* has a high predatory capacity on *S. frugiperda* of 1st and 2nd instar, being able to consume an average of 15 larvae/per day [5]. However, it can consume up to 42 1st instar larvae [15]. For an integrated control, the choice of selective pesticides that do not cause great damage to the predator is of great importance, the product LUFENURON NORTOX 100 EC used in the chemical control of *S. caterpillars frugiperda* is said to be harmless to the predator *D. luteipes* [7], proving to be more appropriate in integrated control programs due to its greater selectivity. The recommended concentration of the insecticide is 150 mg/L in water.

The prey population captured in an interval of time T is the result of the predator’s functional response to prey. There are four types of functional responses, being mainly determined by the density of prey and a set of components that include the time that the predator and the prey remain exposed to each other, the time of the predator searching for the prey, the instant attack rate, counting only the successful ones, the search efficiency, complement the attack rate and the handling time [9]. The Type II response is more frequently observed in invertebrates, being

verified in most insects. In [15] estimated the functional response of the earwigs *D. luteipes* on caterpillars evaluating the potential value of predation. A Type II functional response is described by

$$x_a = \frac{aTx_t}{1 + aT_hx_t} \tag{3.1}$$

where the estimated parameters: $a = 0.3740.055$ -proportion of successful prey attacks per unit; $Th = 182.914.7s$ -prey handling time by the predator, $T = 7200s$ total time of exposure of the predator to prey, x_a -number of 1st instar caterpillars consumed and x_t -density of caterpillars.

By (3.1) and the parameter values, the amount of non-predated caterpillars is given by:

$$N = x_t - x_a = x_t - \frac{aTx_t}{1 + aT_hx_t} \Rightarrow N(x) = x - \frac{aTx}{1 + aT_hx}, \tag{3.2}$$

with $x_t = x$.

In which, the term $\frac{aTx}{1+aT_hx} \Rightarrow \frac{aT}{\frac{1}{x}+aT_h}$, as x increases, $\frac{1}{x} \rightarrow 0$. Thus, it follows that $\frac{aT}{aT_h} = 39.37$ represents the satisfaction value of the scissors *D. luteipes* [15].

The amount of pesticides recommended by the manufacturer depends on the density of prey (x) and predators (P) per area with 10 plants, and it is given by

$$q(x, P) = \frac{q_r}{q_t}N_l \Rightarrow q(x, P) = 0,03(0,9987x - 39,37P) \tag{3.3}$$

where q_r is the amount of pesticide required per 10 plants according to the pest density and assuming maximum application efficiency (solution volume $7\mu L$ /caterpillar), and a lethal concentration for mortality of more than 90% of the pest population, given by $CL > 90 = 15mg/l$ [16]; q_t is the amount recommended by the manufacturer per 10 plants; N_l is the linearization of (3.2) considering the satisfaction level of the predator *D. luteipes* 39.37 caterpillars [10]. The problem can be formulated as the following fuzzy linear programming problem:

$$\min \quad q = 0.03(0.9987x - 39.37P) \tag{3.4}$$

$$\text{subject to :} \quad 0.9987x - 39.37P \leq B_1 \tag{3.5}$$

$$P \geq B_2 \tag{3.6}$$

$$x \geq 0 \tag{3.7}$$

where B_1 and B_2 are fuzzy numbers determined according to economic damage thresholds to *S. frugiperda* and predatory ability of *D. luteipes*. In [15], thresholds for the level of economic damage for *S. frugiperda* in corn were estimated under an area with 10 plants, which is the maximum quantity of caterpillars per plants that cause productivity loss, and that leads to entry with control given by 2.6 and 1.9 larvae for 10 plants in early stages, approximately. The predator is found throughout the year in the field and the presence up to 70% would be enough to keep the pest under control. We considered 10 maize plants. In this way,

$$B_1 = \begin{cases} 1 & , x \leq 30 \\ \frac{100-x}{70} & , 30 < x \leq 100 \\ 0 & , \text{otherwise} \end{cases} \tag{3.8}$$

and

$$B_2 = \begin{cases} 1 & , P \leq 1 \\ \frac{3-P}{2} & , 1 < P \leq 3 \\ 0 & , \text{otherwise} \end{cases} \quad (3.9)$$

where x is the density of prey and P is the predators per area with 10 plants.

The lower bound of ideal values, z_l , is obtained by solving the FLP,

$$\min \quad q = 0.03(0.9987x - 39.37P) \quad (3.10)$$

$$\text{subject to :} \quad 0.9987x - 39.37P \leq 30 \quad (3.11)$$

$$P \geq 1 \quad (3.12)$$

$$x \geq 0 \quad (3.13)$$

As well, the upper limit z_s ,

$$\min \quad q = 0.03(0.9987x - 39.37P) \quad (3.14)$$

$$\text{subject to :} \quad 0.9987x - 39.37P \leq 100 \quad (3.15)$$

$$P \geq 3 \quad (3.16)$$

$$x \geq 0 \quad (3.17)$$

The values obtained were $z_l = 0,8958$ and $z_s = 2.9867$. So, the fuzzy set of optimal values, G , is given by:

$$G = \begin{cases} 1 & , 2.9867 \leq 0.03(0.9987x - 39.37P) \\ \frac{0.03(0.9987x - 39.37P) - 0.8958}{2.0909} & , 0.8958 < 0.03(0.9987x - 39.37P) \leq 2.9867 \\ 0 & , 0.03(0.9987x - 39.37P) \leq 0.8958 \end{cases} \quad (3.18)$$

Now, the fuzzy linear programming problem becomes the following classical optimization problem:

$$\min \quad \lambda \quad (3.19)$$

$$\text{subject to :} \quad 2.0909\lambda - 0.03(0.9987x - 39.37P) \leq -0.8958 \quad (3.20)$$

$$70\lambda + 0.9987x \leq 100 \quad (3.21)$$

$$2\lambda - 39.37P \leq 3 \quad (3.22)$$

$$x, P \geq 0 \quad (3.23)$$

$$0 < \lambda \leq 1 \quad (3.24)$$

The problem has been modelled via Dual Simplex Method in MATLAB.

In addition, the sensitivity analysis is an important step in solving linear programming problems as it allows for evaluating the impact of changes in the problem data on the optimal solution. This

is useful because often the problem conditions may change over time, or may contain uncertainties that affect the parameter values. It allows the decision maker to evaluate how the optimal solution changes in response to changes in the values of the objective function coefficients and problem constraints [13].

Sensitivity analysis using the so-called shadow price is one of the most common techniques for evaluating the influence of constraints on the optimal solution of a linear programming problem. The shadow price is a value that indicates how much the optimal value of the objective function is affected by a change in the right-hand side of a constraint.

The shadow price is an important measure because it allows for evaluating the importance of each constraint to the optimal solution. If a constraint has a high shadow price, it means that the constraint is crucial to the solution and that small changes to the constraint may result in large changes to the solution. Additionally, the shadow price can be used to identify weaknesses in the problem or solution and to evaluate the stability of the solution.

4 RESULTS AND CONCLUSIONS

Solving the classical linear program problem in (3.19), we found the minimum value of $\lambda = 0.7$ for $x = 78.99$. Thus, the amount of pesticides was $q = 0.9162$, that is, this model suggests that the quantity of insecticide was 91.62% of the quantity recommended by manufacturer in the presence of predators.

Using the Dual Simplex Method in MATLAB, it is possible to calculate the shadow price by returning additional information about the sensitivity of the solution. The presented linear programming problem presented in (3.19) a shadow price value of 0 for each constraint, indicating that the constraints are not affecting the optimal solution, i.e., they are not limiting the solution. In this case, the optimal solution can be considered robust with respect to these constraints.

However, it is important to remember that sensitivity analysis should be performed with care and considering all aspects of the problem, and not just the shadow price values. Additionally, it may be that other constraints or variations in the objective function affect the optimal solution more significantly. In this work, our efforts are restricted to this point of sensitivity analysis, and additional items may be discussed in future works

The concern about the environment has been increasingly important. Currently, actions aimed at sustainable management of natural resources are goals. The abusive use of insecticides may lead to an increase number of pests because pests become more resistant, requiring stronger pesticides that will damage the environment even more and will kill the pests' natural predators.

In this way, the model suggests that the quantity of insecticide may be lower than the quantity recommended by manufacturer. Besides, low-quantities of insecticides from that recommended by the manufacturer may be effective. On one hand, there are the costs for each application. On the other hand, there should be a concern about the environmental damage caused by the abusive use of insecticides in the plantations.

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