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Analysis of the Parameters Associated to the Numerical Simulation of the Heat Transfer Process in Agricultural Products

M. AMENDOLA[1], Departamento de Planejamento e Desenvolvimento Rural Sustentável, Faculdade de Engenharia Agrícola, UNICAMP, 13081-970 Campinas, SP, Brazil.

Abstract. The objective of this research is to show the results of the investigation carried out to determine the heat transfer convective coefficient value of different spherical agricultural products, so as to establish and interpret other parameters capable of generating information not only for the product cooling process, but also for characterizing properties and parameters.

1. Introduction

Knowing the cooling time, as well as the parameters that characterize the cooling process of the agricultural products, has become important in supporting the implantation of technologies, which will guarantee the conservation of the same. This experimental research area has shown improvements all over the world, as can be found in specific literature for post-harvest technology.

Particularly at College of Agricultural Engineering at University of Campinas, located in Brazil and known as FEAGRI/UNICAMP, pioneer-research work has been carried out using also applied mathematics as an important tool by de Castro and Amendola [5], Pirozzi [6], Amendola and Teruel [2], and Amendola [1], whose results stimulate the use of the same tool, which establishes the objective of this work. Notice that these researches were carried out in order to compare numerical data with experimental data which were previously obtained as described in these bibliographical references or given by experimental researchers that are still recording them.

2. Material and Methods

For the established objective the mathematical model based on Fourier's second law, adapted from [4] according to the descriptions found in [5], was considered as defined by the equation

[1]E-mail: amendola@agr.unicamp.br

$$\frac{\partial T}{\partial t}(r,t) = \alpha\left(\frac{2}{r}\frac{\partial T}{\partial r}(r,t) + \frac{\partial^2 T}{\partial r^2}(r,t)\right); \quad t \geq 0, \quad r \in [0.R],$$

where

α is the thermal diffusivity of the product $\left(\alpha = \frac{k_p}{\rho C_p}\right)$;
k_p is the thermal conductivity of the product $[\mathrm{W\,m^{-1}K^{-1}}]$;
C_p is the heat capacity of the product $[\mathrm{J\,kg^{-1}K^{-1}}]$;
ρ is the density of the product $[\mathrm{kg\,m^{-3}}]$;
r is the spherical radial coordinate [m];
R is the radius of the product [m];
$T = T(r,t)$ is the temperature of the product [K];
t is the time [s],

and by the initial and boundary conditions, based on the experimental procedure:

$$T(r,0) = T_0; \quad r \in [0,R];$$

$$\frac{\partial T}{\partial r}(0,t) = 0; \quad t \geq 0;$$

and

$$-k_p\frac{\partial T}{\partial r}(R,t) = hc[T_s(t) - T_a(t)]; \quad t \geq 0,$$

where hc is the convective heat transfer coefficient $[\mathrm{W\,m^{-2}\,K^{-1}}]$.

For a resolution of this unidimensional, partial differential equation written in spherical coordinates, a computer program based on the finite differences method according to the implicit scheme of the finite differences method as in [9] was developed. According to the convention $T_i^n = T(i\Delta, n\Delta t)$, for $i = 1,\ldots,N_x$ and for $n = 1,\ldots,N_t$, where N_x defines the spatial mesh and N_t the time mesh, the following approximations were considered:

$$\left.\frac{\partial T}{\partial t}(r,t)\right|_i^n \cong \left(\frac{T_i^{n+1} - T_i^n}{\Delta t}\right); O(\Delta t),$$

$$\left.\frac{\partial^2 T}{\partial r^2}(r,t)\right|_i^{n+1} \cong \left(\frac{T_{i-1}^{n+1} - T_i^{n+1} + T_{i+1}^{n+1}}{\Delta r^2}\right); O(\Delta r^2),$$

$$\left.\frac{\partial T}{\partial r}(r,t)\right|_i^{n+1} \cong \frac{1}{i\Delta r}\left(\frac{T_{i=1}^{n+1} - T_i^{n+1}}{\Delta r}\right); O(\Delta r).$$

Approximations of the same order were considered to the initial and boundary conditions.

The resulting program, implemented in the MATLAB 6.1 environment, was evaluated for the case of numerical simulation of the orange cooling process, which was carried out under determined experimental conditions, for specific values for the thermal properties and parameters as referred in [2]; used in the numeric simulation of the lemon cooling process, the experiment of which was carried out under analogue conditions (unpublished), for the same thermal properties and specific parameters as referred in [1]; and nowadays for pumpkin, for other parameter values, the results of which appear only in this work.

In the case of the pumpkin, the geometry of which is different to the sphere, it was considered that the same was composed of two spherical parts with different radii, and in this work, as a simplified way to study them with the same methodology, only the part with the greater radius was used (unpublished). The thermal constants, which in the cases of the lemon and the orange were taken as being $k = 0.5 \mathrm{W/m\,°C}$ and $\alpha = 1.0600 \times 10^{-7} \mathrm{m^2/s}$, in this case are $k = 1.51125 \mathrm{W/m\,°C}$ and $\alpha = 0.3866862 \times 10^{-6} \mathrm{m^2/s}$.

For each product, the program was carried out for different heat transfer convective coefficient values, as suggested in the literature, here called hc, until the results, when compared with the respective experimental data curve, determined the best adjusted hc value. This adjustment was carried out according to the smallest residue obtained between the experimental and theoretical curves with the use of the Least Squares Method.

Table 1 shows some parameters as well as the results of the hc value obtained so far. In Table 1, the third and fourth columns show the initial and final temperature of the product, as well as the necessary time to reach this last temperature, respectively. In this same table, the last column shows the hc values for each product where the first value is the one obtained by this method with error 2.9% [2]. The second and third values between parenthesis are those obtained by the finite volume method with 7.78% [8] and the finite elements method with 1% [7]. These differences probably are due to the mathematical model considered, one-dimensional only in the first case, and also due to the spatial mesh size: 128, 20×20 and 2521×4861 respectively.

Table 1: Conditions and parameters considered and obtained in the experiments and theoretical results for the hc values for different products.

Product	Radius r (m)	Ti (℃)	Tf (℃) ; tf (min)	hc ($\mathrm{W/m^2\,°C}$)
Orange	0.038	26.57	1.3; 150	73 (56.48; 64)
Lemon	0.0125	27.23	2.8; 50	12.25
Pumpkin	0.045	26.90	3.21; 170	13.52

Analyzing the theory associated to the product cooling process, one can understand, for instance, that the hc reflects the intensity of the process, and thus, at a first view, the hc value for the lemon should be higher when compared to the value obtained for the orange, as the experiments were carried out under the same conditions and the process of the former took 1/3 of the time of the latter. This observation induces mathematical analyses, such as the ones that follow.

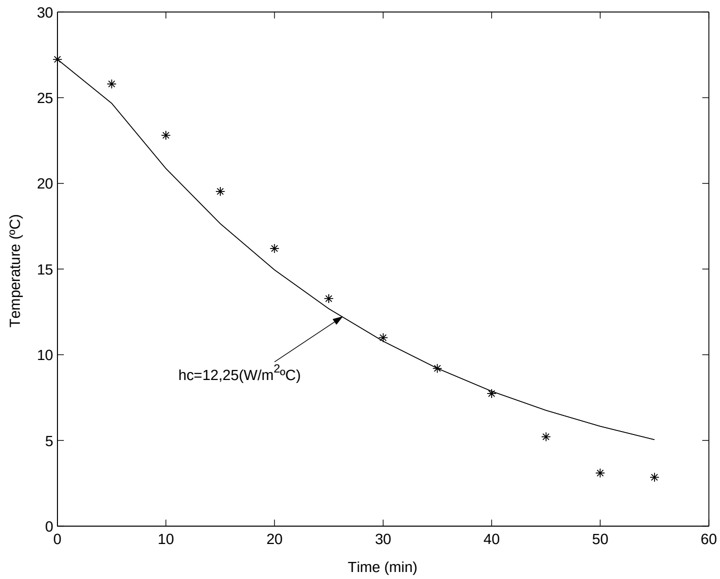

Figure 1: Experimental data of lemon cooling, collected every 5 minutes, and those simulated with the hc adjusted values = 12.25W/m ℃.

3. Results and Discussions

Above all, it should be emphasized that the fact that different hc values were found for each one of the distinct numeric methods used in the case of the orange (last column of Table 1), reveals the necessity of continuing investigations and mathematical analyses for an effective hc value. Apart from this, the results shown below are taken from the use of the finite differences method.

A first approach was the analysis of the experimental data of the lemon, as all the calculations carried out up to the point, for all products, were based on the experimental data collected every 5 minutes. However, only the lemon had data collected every minute as well.

These values as well as the adjustments carried out and the respective adjusted hc values, in which fortunately the same values were obtained, are presented in Figures 1 and 2.

However, as certain instability can be noticed, due to the data not following the theoretically smooth behavior expected, new simulations were carried out, modifying the group of data according to a visual criteria, from which different hc values were obtained, as presented in Figures 3 and 4.

These results confirm the obvious and high sensitivity of the hc value as a function of a subgroup of the experimental data, pointing out the experimental process as being responsible for the accuracy of the theoretical results obtained. Apart from that, it reinforces the importance of the consideration of the initial and final times of the experiment.

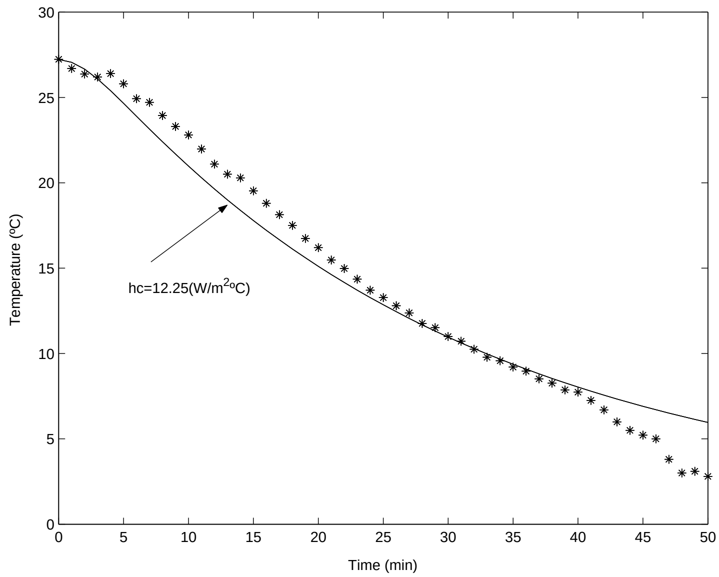

Figure 2: Experimental data of the lemon cooling, collected every minute, and those simulated with the hc adjusted values = 12.25W/m ℃.

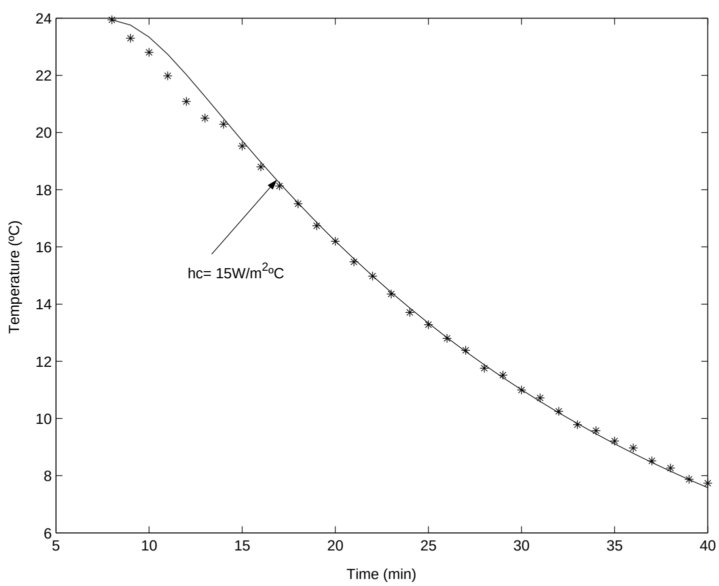

Figure 3: Experimental data of the lemon cooling, collected every minute, starting 8 minutes after the beginning of the experiment until 40 minutes after, and those simulated with the hc adjusted values = $15\mathrm{W/m}^2$ ℃.

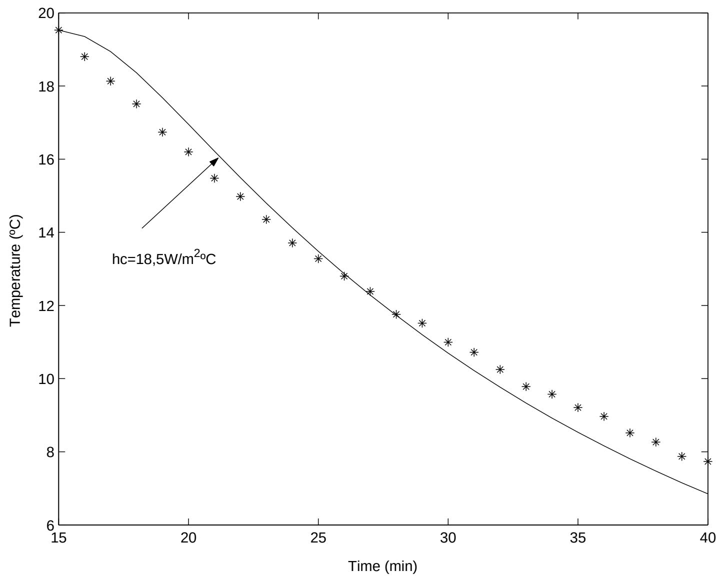

Figure 4: Experimental data of the lemon cooling, collected every minute, starting 15 minutes after the beginning of the experiment until 40 minutes after, and those simulated with the hc adjusted values = $18.5 \mathrm{W/m^2\,℃}$.

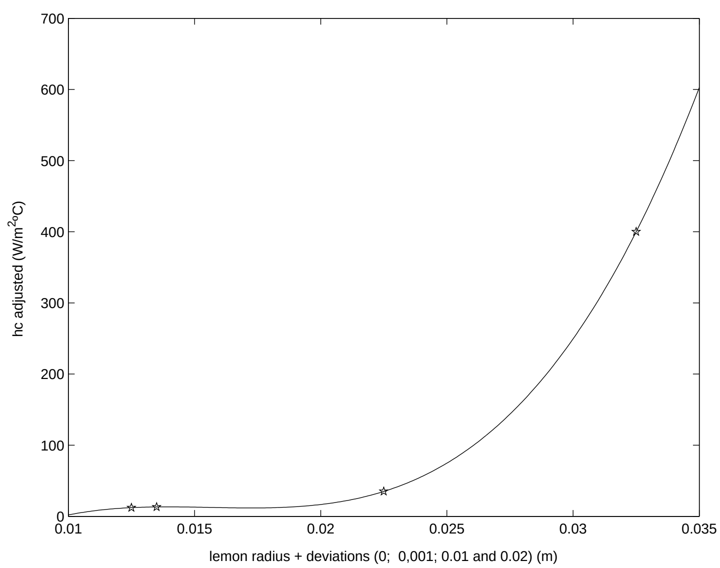

Figure 5: Fictitious adjusted hc values for the lemon, as a function of the deviation of the real radius of the same.

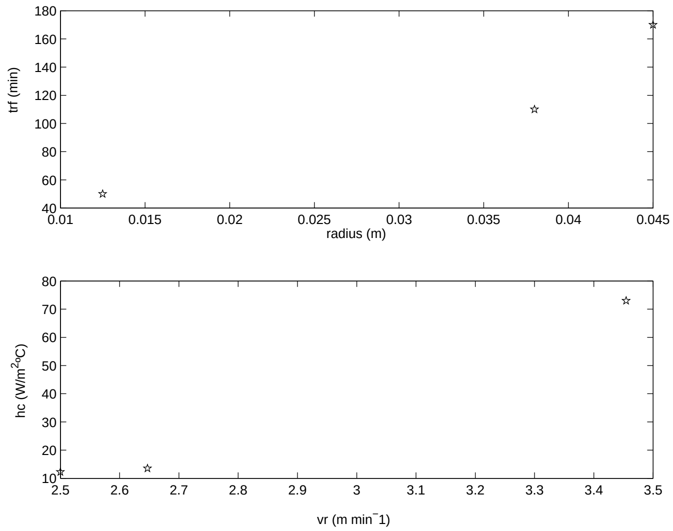

Figure 6: A) Time after which each product reaches a temperature of 3℃ due to the radius of the product. B) hc value due to the vr value for each product.

In addition to this, simulations of the cooling process, considering distinct deviations of the radius' real value, denoted by r1, were carried out for the lemon. The r2 = r1 + 0.001; r3 = r1 + 0.01 and r3 = r1 + 0.02 values were specifically arbitrarily considered and the same kind of adjustment for the determination of the hc value was carried out, as shown in Figure 5. In this Figure, the line is presented continuous only to facilitate the interpretation of the data.

These fictitious values, thus called, due to not being considered real experiments, show a strong influence of the radius value of the product in the hc value, as well as the consequent hc variation, due to the localization of the thermopairs. This induces another type of mathematical analysis, simple though real, carried out in an attempt to investigate the relation between the hc value and the ratio value, called vr, established between the product's radius value and its final cooling time.

As this investigation was carried out for all the products, the referred final time (tfr) was considered as the time after which the products reached a same temperature of approximately 3 ℃. The value of the adjusted hc and the vr = r/tfr ratio for each product are presented in Table 2.

Table 2: hc values (W/m^2 ℃) and vr = r/tf(m/min) ratio for the different products.

Product	hc(W/m^2 ℃)	vr(m/min)= r/tfr
Orange	73	3.4545×10^{-4}
Lemon	12.25	2.5000×10^{-4}
Pumpkin	13.52	2.6471×10^{-4}

In Table 2, it can be seen that the obtained vr values for each product, reveal a new factor for the interpretation of the cooling process, as hc is directly proportional to vr, being therefore, considered a parameter that translates the speed of the process, validating the obtained hc values, and interpreted in this way. For a better comprehension of these results, graphs, which appear in Figure 6, were elaborated.

These results reveal that the vr parameter characterizes the speed of the cooling process of each product, and therefore can be used not only to characterize the product but also to support the decision to implant post-harvest technology.

4. Conclusions

Similarly to what was concluded in previous research, it is reinforced that numerical simulation and mathematical analysis are adequate tools for the completion of this investigation, the results of which can only be conclusive after precise considerations on the origin and representativity of the experimental data, and that, therefore, there is still the need for the continuation of this kind of research, looking for to be able to join the experimental and theoretical tools used.

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