Submetido para TEMA

Additional constraints to ensure three vanishing moments for orthonormal wavelet filter banks and transient detection

Abstract. This article presents an improvement to the formulation of Sherlock and Monro for the wavelet parameterization for the obtainment of the restrictions which ensure three vanishing moments. In order to test the formulation presented, a transient signal detection is presented.

Keywords. Vanishing Moments, Additional Constraints, Transient detection.

1. Introduction

Sherlock and Monro [7] started the study of the angular parameterization of orthonormal filter banks, adapting the work of [9] on the factorization of paraunitary matrices and parameterizing the space of orthonormal wavelets by a set of angular parameters.

Initially the formulation had a weak point, there were no restrictions to ensure a number of vanishing moments greater than one. Additional restrictions to ensure at least two vanishing moments were obtained by [5]. This article is an extension of [8] that presents an improvement to the formulation of [7], in order to ensure a third vanishing moment for wavelet filter banks, additional constraints are presented to the work of [5] and [7]. An application of this formulation with three vanishing moments for transient detection is presented in this paper.

Let

$$H^{(N)}(z) = \sum_{i=0}^{2N-1} h_i^{(N)} z^{-i}, \quad G^{(N)}(z) = \sum_{i=0}^{2N-1} g_i^{(N)} z^{-i}$$

be the transfer functions of the lowpass and highpass filters, respectively, for an orthonormal filter bank with length-$2N$, where

$$g_i^{(N)} = (-1)^{i+1} h_{2N-1-i}^{(N)}, \qquad i = 0, 1, \cdots, 2N-1, \tag{1.1}$$

and

$$
\begin{aligned}
h_0^{(1)} &= \cos(\alpha_1)\\
h_1^{(1)} &= \sin(\alpha_1)
\end{aligned}
$$

$$
\begin{aligned}
h_0^{(N+1)} &= \cos(\alpha_{N+1})h_0^{(N)}\\
h_{2i}^{(N+1)} &= \cos(\alpha_{N+1})h_{2i}^{(N)} - \sin(\alpha_{N+1})h_{2i-1}^{(N)}\\
h_{2N}^{(N+1)} &= -\sin(\alpha_{N+1})h_{2N-1}^{(N)}\\
h_1^{(N+1)} &= \sin(\alpha_{N+1})h_0^{(N)}\\
h_{2i+1}^{(N+1)} &= \sin(\alpha_{N+1})h_{2i}^{(N)} + \cos(\alpha_{N+1})h_{2i-1}^{(N)}\\
h_{2N+1}^{(N+1)} &= \cos(\alpha_{N+1})h_{2N-1}^{(N)}
\end{aligned}
\qquad i = 0, 1, \cdots, 2N-1. \tag{1.2}
$$

2. First and second vanishing moments

If the filter bank is to characterize a wavelet transform, the regularity condition $G^{(N)}(z)|_{z=1} = 0$ must be satisfied [1, 4]. Which, according to [5], leads to

$$\alpha_N = \frac{\pi}{4} - \sum_{i=1}^{N-1} \alpha_i. \tag{2.1}$$

According to [4], to ensure two vanishing moments it is necessary that $\left.\frac{dG^{(N)}(z)}{dz}\right|_{z=1} = 0$. This provides, [5],

$$\alpha_{N-1} = \frac{1}{2}\arcsin\left\{-\frac{1}{2} - \sum_{i=1}^{N-2}\left[\sin\sum_{i=1}^{k} 2\alpha_i\right]\right\} - \sum_{i=1}^{N-2}\alpha_i. \tag{2.2}$$

Equation (2.2) has a real solution if the angles α_i satisfy the condition

$$-\frac{3}{2} \leq \sum_{i=1}^{N-2}\left[\sin\sum_{i=1}^{k} 2\alpha_i\right] \leq \frac{1}{2}. \tag{2.3}$$

3. The Third Vanishing Moment

In order to obtain a third vanishing moment, [8], it is necessary that

$$\left.\frac{d^2G^{(N)}(z)}{dz^2}\right|_{z=1} = 0. \tag{3.1}$$

Replacing (1.1) in the second derivative of $G^{(N)}(z)$ when $z = 1$ and writing conveniently becomes

$$\left.\frac{d^2G^{(N)}(z)}{dz^2}\right|_{z=1} = \begin{array}{l} (N^2+2N-1)\sum_{i=0}^{N-1} h_{2i}^{(N)} + \\ \sum_{i=1}^{N-1}(2N+1)\left[\sum_{i=0}^{N-1} h_{2i}^{(N)} - 2\sum_{i=N-j+1}^{N-1} h_{2i}^{(N)}\right] + \\ \sum_{j=1}^{N-2}\sum_{k=1}^{N-j-1} 2\left[\sum_{i=0}^{N-1} h_{2i}^{(N)} - 2\sum_{i=k+1}^{k+j} h_{2i}^{(N)}\right] - \\ (N^2-1)\sum_{i=0}^{N-1} h_{2i+1}^{(N)} + \\ \sum_{i=1}^{N-1}(2N-1)\left[\sum_{i=0}^{N-1} h_{2i+1}^{(N)} - 2\sum_{i=N-j+1}^{N-1} h_{2i+1}^{(N)}\right] - \\ \sum_{j=1}^{N-2}\sum_{k=1}^{N-j-1} 2\left[\sum_{i=0}^{N-1} h_{2i+1}^{(N)} - 2\sum_{i=k+1}^{k+j} h_{2i+1}^{(N)}\right]. \end{array} \tag{3.2}$$

Lemma 3.1. *Considering (1.2), the following equalities are true*

$$\sum_{i=0}^{N-1} h_{2i}^{(N)} = \cos\left[\sum_{i=1}^{N} \alpha_i\right], \tag{3.3}$$

$$\sum_{i=0}^{N-1} h_{2i+1}^{(N)} = \sin\left[\sum_{i=1}^{N} \alpha_i\right]. \tag{3.4}$$

Proof. Proof by induction:

In the case that $N = 1$, to (3.3) and (3.4), respectively, it has

$$h_0^{(1)} = \cos(\alpha_1) \text{ and } h_1^{(1)} = \sin(\alpha_1).$$

Demonstrate that if

$$\sum_{i=0}^{N-1} h_{2i}^{(N)} = \cos\left[\sum_{i=1}^{N} \alpha_i\right], \tag{3.5}$$

$$\sum_{i=0}^{N-1} h_{2i-1}^{(N)} = \sin\left[\sum_{i=1}^{N} \alpha_i\right] \tag{3.6}$$

then

$$\sum_{i=0}^{N+1} h_{2i}^{(N+1)} = \cos\left[\sum_{i=1}^{N+1} \alpha_i\right], \tag{3.7}$$

$$\sum_{i=0}^{N+1} h_{2i+1}^{(N+1)} = \sin\left[\sum_{i=1}^{N+1} \alpha_i\right] \tag{3.8}$$

a) Demonstrate that the validity of (3.5) and (3.6) implies the validity of (3.7):

$$\begin{aligned}
\sum_{i=0}^{N+1} h_{2i}^{(N+1)} &= \sum_{i=0}^{N}\left[\cos(\alpha_{N+1})h_{2i}^{(N)} - \sin(\alpha_{N+1})h_{2i-1}^{(N)}\right] \\
&= \cos(\alpha_{N+1})\sum_{i=0}^{N}\left[h_{2i}^{(N)}\right] - \sin(\alpha_{N+1})\sum_{i=0}^{N}\left[h_{2i-1}^{(N)}\right] \\
&= \cos(\alpha_{N+1})\cos\left[\sum_{i=1}^{N}\alpha_i\right] - \sin(\alpha_{N+1})\sin\left[\sum_{i=1}^{N}\alpha_i\right] \\
&= \cos\left[\alpha_{N+1} + \sum_{i=1}^{N}\alpha_i\right] = \cos\left[\sum_{i=1}^{N+1}\alpha_i\right]
\end{aligned}$$

b) Demonstrate that the validity of (3.5) and (3.6) implies the validity of (3.8):

$$\begin{aligned}
\sum_{i=0}^{N+1} h_{2i+1}^{(N+1)} &= \sum_{i=0}^{N}\left[\sin(\alpha_{N+1})h_{2i}^{(N)} + \cos(\alpha_{N+1})h_{2i-1}^{(N)}\right] \\
&= \sin(\alpha_{N+1})\sum_{i=0}^{N}\left[h_{2i}^{(N)}\right] + \cos(\alpha_{N+1})\sum_{i=0}^{N}\left[h_{2i-1}^{(N)}\right] \\
&= \sin(\alpha_{N+1})\cos\left[\sum_{i=1}^{N}\alpha_i\right] + \cos(\alpha_{N+1})\sin\left[\sum_{i=1}^{N}\alpha_i\right] \\
&= \sin\left[(\alpha_{N+1}) + \sum_{i=1}^{N}\alpha_i\right] = \sin\left[\sum_{i=1}^{N+1}\alpha_i\right]
\end{aligned}$$

□

Lemma 3.2. *The equations in (1.2) imply*

$$\sum_{i=1}^{N-1}\left[\sum_{i=0}^{k} h_{2i}^{(N)} - \sum_{i=0}^{N-1} h_{2i}^{(N)}\right] = \sum_{i=1}^{N-1}\cos\left[\sum_{i=1}^{k}\alpha_i - \sum_{i=k+1}^{N}\alpha_i\right], \tag{3.9}$$

$$\sum_{i=1}^{N-1}\left[-\sum_{i=0}^{k} h_{2+i}^{(N)} + \sum_{i=0}^{N-1} h_{2+i}^{(N)}\right] = \sum_{i=1}^{N-1}\sin\left[\sum_{i=1}^{k}\alpha_i - \sum_{i=k+1}^{N}\alpha_i\right]. \tag{3.10}$$

Proof. For $N = 1$ the verification of the validity of (3.9) and (3.10) is immediate.
For $N > 1$ show that if

$$\sum_{i=1}^{N-1}\left[\sum_{i=0}^{k} h_{2i}^{(N)} - \sum_{i=k+1}^{N-1} h_{2i}^{(N)}\right] = \sum_{i=1}^{N-1}\cos\left[\sum_{i=1}^{k}\alpha_i - \sum_{i=k+1}^{N}\alpha_i\right] \tag{3.11}$$

then

$$\sum_{i=1}^{N+1}\left[\sum_{i=0}^{k} h_{2i}^{(N+1)} - \sum_{i=k+1}^{N+1} h_{2i}^{(N+1)}\right] = \sum_{i=1}^{N+1}\cos\left[\sum_{i=1}^{k}\alpha_i - \sum_{i=k+1}^{N+2}\alpha_i\right], \tag{3.12}$$

and if

$$\sum_{i=1}^{N-1}\left[-\sum_{i=0}^{k} h_{2+i}^{(N)} + \sum_{i=0}^{N-1} h_{2+i}^{(N)}\right] = \sum_{i=1}^{N-1}\sin\left[\sum_{i=1}^{k}\alpha_i - \sum_{i=k+1}^{N}\alpha_i\right] \tag{3.13}$$

then

$$\sum_{i=1}^{N+1}\left[-\sum_{i=0}^{k} h_{2+i}^{(N+1)} + \sum_{i=0}^{N+1} h_{2+i}^{(N+1)}\right] = \sum_{i=1}^{N+1}\sin\left[\sum_{i=1}^{k}\alpha_i - \sum_{i=k+1}^{N+2}\alpha_i\right]. \tag{3.14}$$

Demonstrate that the validity of (3.11) implies the validity of (3.12):

$$\begin{aligned}
\sum_{i=1}^{N+1}\left[\sum_{i=0}^{k} h_{2i}^{(N+1)} - \sum_{i=k+1}^{N+1} h_{2i}^{(N+1)}\right] &= \sum_{i=1}^{N+1}\left[\sum_{i=0}^{k}\left[\cos(\alpha_{N+1})h_{2i}^{(N)} - \sin(\alpha_{N+1})h_{2i-1}^{(N)}\right] - \right.\\
&\quad \left.\sum_{i=k+1}^{N+1}\left[\cos(\alpha_{N+1})h_{2i}^{(N)} - \sin(\alpha_{N+1})h_{2i-1}^{(N)}\right]\right]\\
&= \sum_{i=1}^{N+1}\left[\cos(\alpha_{N+1})\sum_{i=0}^{k} h_{2i}^{(N)} - \sin(\alpha_{N+1})\sum_{i=0}^{k} h_{2i-1}^{(N)} - \right.\\
&\quad \left.\cos(\alpha_{N+1})\sum_{i=k+1}^{N+1} h_{2i}^{(N)} + \sin(\alpha_{N+1})\sum_{i=k+1}^{N+1} h_{2i-1}^{(N)}\right].
\end{aligned}$$

From Lemma 3.1 it follows that

$$\begin{aligned}
\sum_{i=1}^{N+1}\left[\sum_{i=0}^{k} h_{2i}^{(N+1)} - \sum_{i=k+1}^{N+1} h_{2i}^{(N+1)}\right] &= \sum_{i=1}^{N+1}\left[\cos(\alpha_{N+1})\cos\sum_{i=0}^{k}\alpha_i - \sin(\alpha_{N+1})\sin\sum_{i=0}^{k}\alpha_i - \right.\\
&\quad \left.\cos(\alpha_{N+1})\cos\sum_{i=k+1}^{N+1}\alpha_i + \sin(\alpha_{N+1})\sin\sum_{i=k+1}^{N+1}\alpha_i\right]\\
&= \sum_{i=1}^{N+1}\left[\cos\left(\alpha_{N+1} + \sum_{i=0}^{k}\alpha_i\right) - \cos\left(\alpha_{N+1} + \sum_{i=k+1}^{N+1}\alpha_i\right)\right]\\
&= \sum_{i=1}^{N+1}\cos\left[\sum_{i=1}^{k}\alpha_i - \sum_{i=k+1}^{N+2}\alpha_i\right].
\end{aligned}$$

Demonstrate that the validity of (3.13) implies the validity of (3.14):

$$\sum_{i=1}^{N-1}\left[-\sum_{i=0}^{k} h_{2+i}^{(N)} + \sum_{i=0}^{N-1} h_{2+i}^{(N)}\right] = \sum_{i=1}^{N-1}\left[-\sum_{i=1}^{k}\left[\sin(\alpha_{N+1})h_{2i}^{(N)} + \cos(\alpha_{N+1})h_{2i-1}^{(N)}\right] + \right.$$

$$
\begin{aligned}
&\sum_{i=k+1}^{k}\left[\sin(\alpha_{N+1})h_{2i}^{(N)}+\cos(\alpha_{N+1})h_{2i-1}^{(N)}\right]\Bigg] \\
=&\sum_{i=1}^{N-1}\Bigg[-\sin(\alpha_{N+1})\sum_{i=1}^{k}h_{2i}^{(N)}-\cos(\alpha_{N+1})\sum_{i=1}^{k}h_{2i-1}^{(N)}+ \\
&\sin(\alpha_{N+1})\sum_{i=k+1}^{k}h_{2i}^{(N)}+\cos(\alpha_{N+1})\sum_{i=k+1}^{k}h_{2i-1}^{(N)}\Bigg].
\end{aligned}
$$

From Lemma 3.1 it follows that

$$
\begin{aligned}
\sum_{i=1}^{N-1}\left[-\sum_{i=0}^{k}h_{2+i}^{(N)}+\sum_{i=0}^{N-1}h_{2+i}^{(N)}\right] &= \sum_{i=1}^{N-1}\Bigg[-\sin(\alpha_{N+1})\cos\sum_{i=0}^{k}\alpha_i-\cos(\alpha_{N+1})\sin\sum_{i=0}^{k}\alpha_i+ \\
&\sin(\alpha_{N+1})\cos\sum_{i=k+1}^{N+1}\alpha_i+\cos(\alpha_{N+1})\sin\sum_{i=k+1}^{N+1}\alpha_i\Bigg] \\
&= \sum_{i=1}^{N+1}\left[-\sin\left(\alpha_{N+1}+\sum_{i=1}^{k}\alpha_i\right)+\sin\left(\alpha_{N+1}+\sum_{i=1}^{k}\alpha_i\right)\right] \\
&= \sum_{i=1}^{N+1}\sin\left[\sum_{i=1}^{k}\alpha_i-\sum_{i=k+1}^{N+2}\alpha_i\right].
\end{aligned}
$$

□

From the lemmas 3.1 and 3.2 it follows that (3.2) can be written as

$$
\left.\frac{d^2G^{(N)}(z)}{dz^2}\right|_{z=1} = \begin{array}{l}(N^2+2N-1)\cos\beta-(N^2-1)\sin\beta+ \\ \displaystyle\sum_{j=1}^{N-1}\left\{(2N+1)\cos\beta_j+(2N-1)\sin\beta_j\right\}+ \\ \displaystyle\sum_{j=1}^{N-2}\left\{\sum_{k=1}^{N-j-1}(2\cos\beta_{j,k}-2\sin\beta_{j,k})\right\}\end{array} \tag{3.15}
$$

where

$$
\begin{cases}
\beta=\displaystyle\sum_{i=1}^{N}\alpha_i \\
\beta_j=\displaystyle\sum_{i=1}^{N}\alpha_i-2\sum_{i=(N-j)+1}^{N}\alpha_i, \qquad 1\le j\le N-1 \\
\beta_{j,k}=\displaystyle\sum_{i=1}^{N}\alpha_i-2\sum_{i=k+1}^{k+j}\alpha_i, \qquad 1\le j\le N-1 \text{ e } 1\le k\le N-1.
\end{cases} \tag{3.16}
$$

From equation (2.1), equation (3.16) has the following implications:

$$\beta = \frac{\pi}{4},\ \beta_j = \frac{\pi}{4} - 2\sum_{i=(N-j)+1}^{N} \alpha_i \text{ e } \beta_{j,k} = \frac{\pi}{4} - 2\sum_{i=k+1}^{k+j} \alpha_i.$$

Leading to some properties:

$$(N^2+2N-1)\cos\beta - (N^2-1)\sin\beta = 2N\cos\frac{\pi}{4},$$

$$\cos\left[\frac{\pi}{4} - 2\sum_{i=(N-j)+1}^{N}\alpha_i\right] + \sin\left[\frac{\pi}{4} - 2\sum_{i=(N-j)+1}^{N}\alpha_i\right] = 2\cos\frac{\pi}{4}\cos\left[2\sum_{i=(N-j)+1}^{N}\alpha_i\right],$$

$$\cos\left[\frac{\pi}{4} - 2\sum_{i=k+1}^{k+j}\alpha_i\right] - \sin\left[\frac{\pi}{4} - 2\sum_{i=k+1}^{k+j}\alpha_i\right] = 2\cos\frac{\pi}{4}\sin\left[2\sum_{i=k+1}^{k+j}\alpha_i\right],$$

$$\cos\left[\frac{\pi}{4} - 2\sum_{i=(N-j)+1}^{N}\alpha_i\right] = \cos\frac{\pi}{4}\left(\cos\left[2\sum_{i=(N-j)+1}^{N}\alpha_i\right] + \sin\left[2\sum_{i=(N-j)+1}^{N}\alpha_i\right]\right).$$

Using these properties, equation (3.15-3.16) can be written as:

$$\left.\frac{d^2G^{(N)}(z)}{dz^2}\right|_{z=1} = \begin{array}{l} 2N\cos\frac{\pi}{4} + \sum_{j=1}^{N-1}\left\{2\cos\frac{\pi}{4}(\cos\lambda_j + \sin\lambda_j) + 2(2N-1)\cos\frac{\pi}{4}\cos\lambda_j\right\} + \\ \sum_{j=1}^{N-2}\left\{\sum_{k=1}^{N-j-1}\left(4\cos\frac{\pi}{4}\sin\lambda_k\right)\right\} \end{array} \tag{3.17}$$

where $\lambda_j = \left[2\sum_{i=(N-j)+1}^{N}\alpha_i\right]$ e $\lambda_k = \left[2\sum_{i=k+1}^{k+j}\alpha_i\right]$.

Applying (3.1) in (3.17)

$$0 = N + \sum_{j=1}^{N-1}\left\{\sin\lambda_j + 2N\cos\lambda_j\right\} + \sum_{j=1}^{N-2}\left\{\sum_{k=1}^{N-j-1}(2\sin\lambda_k)\right\}, \tag{3.18}$$

Then (3.18) should be written in terms of α_{N-2}, firstly rewriting (3.18),

$$\sum_{j=1}^{N-1}\left\{\sin\lambda_j + 2N\cos\lambda_j\right\} + \sum_{j=1}^{N-2}\left\{\sum_{k=1}^{N-j-1} 2\sin\left[\sum_{i=k+1}^{k+j} 2\alpha_i\right]\right\} + N = 0. \tag{3.19}$$

Decomposing the second parcel of (3.19) gives

$$\sum_{j=1}^{N-1}\left\{\sin\lambda_j+2N\cos\lambda_j\right\}+2\sin\left[\sum_{2}^{N-1}2\alpha_i\right]+\sum_{j=1}^{N-3}\left\{\sum_{k=1}^{N-j-1}2\sin\left[\sum_{i=k+1}^{k+j}2\alpha_i\right]\right\}=-N$$

and following the reasoning

$$2\sin\left[\sum_{2}^{N-1}2\alpha_i\right]=-\left\{\sum_{j=1}^{N-1}\left\{\sin\lambda_j+2N\cos\lambda_j\right\}+\sum_{j=1}^{N-3}\left\{\sum_{k=1}^{N-j-1}2\sin\lambda_k\right\}+N\right\}$$
$$\sum_{2}^{N-1}2\alpha_i=\arcsin\left\{-\sum_{j=1}^{N-1}\left\{\frac{1}{2}\sin\lambda_j+N\cos\lambda_j\right\}-\sum_{j=1}^{N-3}\left\{\sum_{k=1}^{N-j-1}\sin\lambda_k\right\}-\frac{N}{2}\right\}$$
$$\sum_{i=2,i\neq N-2}^{N-1}2\alpha_i+2\alpha_{N-2}=\arcsin\left\{-\sum_{j=1}^{N-1}\left\{\frac{1}{2}\sin\lambda_j+N\cos\lambda_j\right\}-\sum_{j=1}^{N-3}\left\{\sum_{k=1}^{N-j-1}\sin\lambda_k\right\}-\frac{N}{2}\right\}$$
$$\alpha_{N-2}=\frac{1}{2}\arcsin\left\{-\sum_{j=1}^{N-1}\left\{\frac{1}{2}\sin\lambda_j+N\cos\lambda_j\right\}-\sum_{j=1}^{N-3}\left\{\sum_{k=1}^{N-j-1}\sin\lambda_k\right\}-\frac{N}{2}\right\}-\sum_{i=2,i\neq N-2}^{N-1}\alpha_i. \tag{3.20}$$

Equation (3.20) ensures the third vanishing moment, but the equation (3.20) has a real solution if the angles α_i satisfy the condition

$$-1-\frac{N}{2}\leq\left\{\sum_{j=1}^{N-1}\left\{\frac{1}{2}\sin\lambda_j+N\cos\lambda_j\right\}+\sum_{j=1}^{N-3}\left\{\sum_{k=1}^{N-j-1}\sin\lambda_k\right\}\right\}\leq 1-\frac{N}{2}. \tag{3.21}$$

4. Transient detection of a signal

Consider a orthonormal filter bank with length-8 ($N=4$), which initial configuration characterizes a wavelet with at least one vanishing moment, $\alpha_{p=1}=\{-17.38°, 16.83°, -45.10°, 90.65°\}$. To ensure two vanishing moments complies (2.2) resulting in $\alpha_{p=2}=\{-17.38°, 16.83°, 3.12°, 42.43°\}$. To ensure at least three vanishing moments apply (3.20) which leads to $\alpha_{p=3}=\{-17.38°, -47.23°, 93.44°, 16.17°\}$. Figure 1 shows the functions $\psi(t)$: $\alpha_{p=1}$, $\alpha_{p=2}$ and $\alpha_{p=3}$, respectively. Each wavelet has the sampling frequency which is denoted by $\omega_s=2\pi/Ts$.

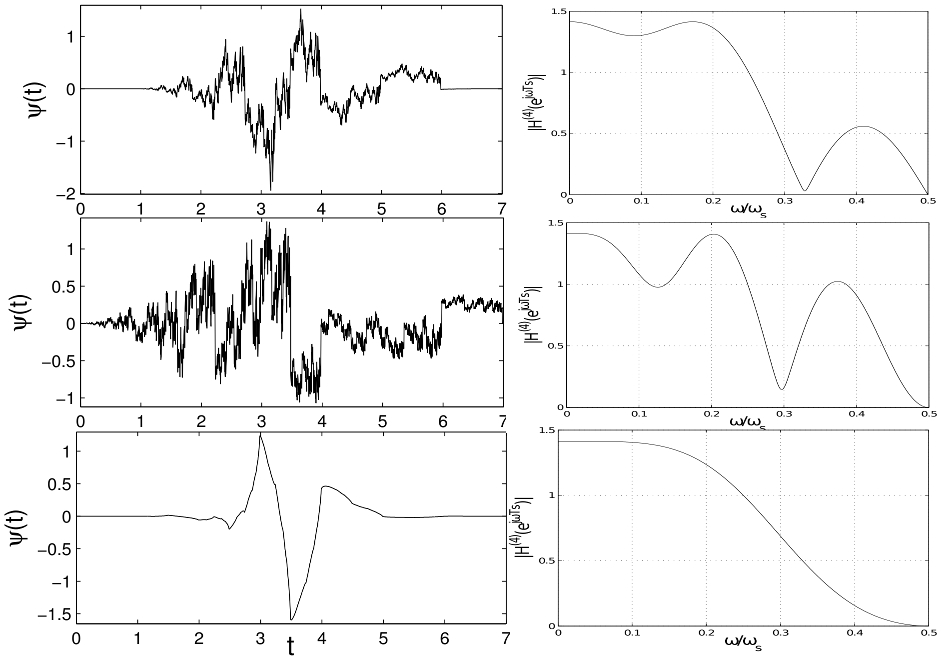

Figure 1: On the left: $\alpha_{p=1}$, $\alpha_{p=2}$ and $\alpha_{p=3}$. On the right: the sampling frequency of each wavelet.

Let $f(t)$ be the signal shown in Figure 2,

$$f(t) = \begin{cases} \dfrac{t}{e^t} & \text{if} \quad 0 \leq t < 377 \\ \dfrac{(t-0.001)}{e^t} & \text{if} \quad 377 \leq t < 995 \\ 0.3683 & \text{if} \quad 995 \leq t \leq 1200. \end{cases}$$

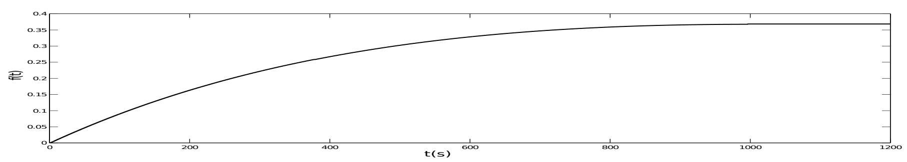

Figure 2: Signal $f(t)$.

This signal has two transients (discontinuities) and was analyzed using $\alpha_{p=1}$, $\alpha_{p=2}$ and $\alpha_{p=3}$, according to figures 3, 4 and 5. Each figure shows the decomposition of the first and second wavelet levels, respectively.

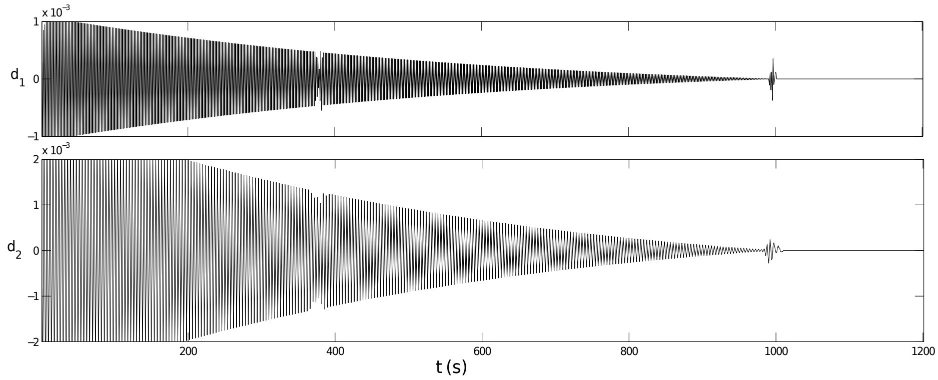

Figure 3: Analysis of $f(t)$ with $\alpha_{p=1}$ in the first and second level of decomposition.

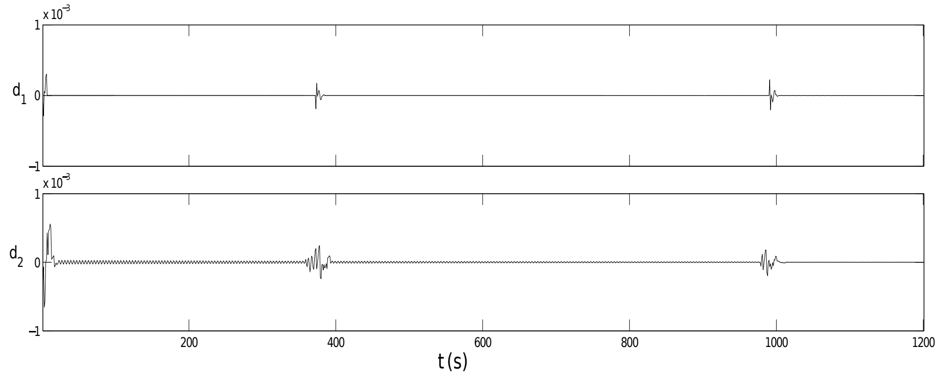

Figure 4: Analysis of $f(t)$ with $\alpha_{p=2}$ in the first and second level of decomposition.

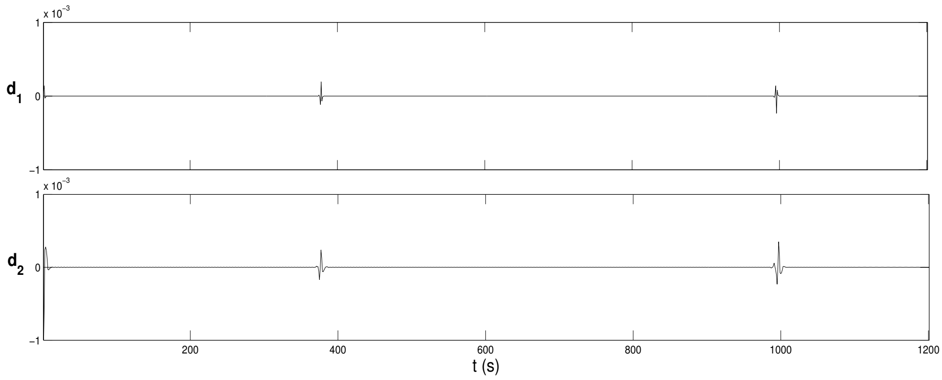

Figure 5: Analysis of $f(t)$ with $\alpha_{p=3}$ in the first and second level of decomposition.

Comparing the figures 3, 4 and 5 it is noticed that despite of the good identification of transients using $\alpha_{p=2}$, the analysis with $\alpha_{p=3}$ also provides a good result, now for $\alpha_{p=1}$ the detection is not quite clear. The amplitude of the detail coefficients, besides the transients that appear in the first decomposition level with $\alpha_{p=2}$, decrease in the case of $\alpha_{p=3}$.The presence of high frequency coefficients indicates that the transients are slightly more highlighted when the analysis is done using $\alpha_{p=3}$.

There are other formulations to work with wavelet filter banks, for example, [10], but the formulation of Sherlock and Monro stands for the mathematical and computational simplicities. However, initially there were no constraints to ensure a number of vanishing moments greater than one. An extension of this formulation introducing restrictions to ensure two vanishing moments was done by [5]. In [8] the constraints to ensure at least three vanishing moments were presented.

Several papers on applications using this formulation before the extension for three vanishing moments have been published, some examples are, such as pattern recognition [2], linear estimation [3], and signal compression [6].

5. Conclusions

This paper presented the constraints that ensure three vanishing moments and also demonstrations and calculations for obtaining the same. I also presents a brief application of the formulation for transient detection in signals, a comparative way between wavelets with different regularities.

In this paper, an application example was used to test the three different wavelets of Sherlock and Monro. Through this example it was noticed that those wavelets are efficient in transient detection, specially when regularity is of at least two vanishing moments. In the case that the parameterization satisfies at least three vanishing moments it was obtained a good identification of transients and better compression or the regular parts of the signal. This fact supports the idea that the more regular is the wavelet the better is the compression of the regular parts of the signal to be decomposed.

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