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SDRE based Leader-Follower Formation Control of Multiple Mobile Robots*

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ABSTRACT. Formation control of multiple mobile robots is a relatively new research area of robotics and increase the control performance and advantages of multiple mobile robots systems. In this work we present a study concerning the modeling and formation control of a robotic system composed by two mobile robots, the leader robot and the follower robot. The system is a nonlinear dynamical system and cannot be controlled by traditional linear control techniques. The control strategy proposed is the SDRE (State-Dependent Riccati Equation) method. Simulations results with the software Matlab show the efficiency of the control method.

Keywords: formation control, multi-robot systems, mobile robots, nonlinear dynamical systems, SDRE control.

1 INTRODUCTION

Formation control of multiple robots have drawn an extensive research attention in robotics and control community recently. The objective of formation control of multiple mobile robots is maintain a desired orientation and distance between two or more mobile robots. In this work we study two mobile robots. This area has a wide range of applications like transportation of large objects [3], surveillance [7], exploration [9], etc. The main advantages of formation control are reliability, adaptability, flexibility and perform complex missions and tasks that would be certainly impracticable for a single mobile robot.

The main approaches and strategies proposed in the literature for the formation control are virtual structure, behavior based and leader-follower [3],[12],[13]. The virtual structure treats the entire formation as a single virtual rigid structure. By behavior based approach, several desired behaviors are prescribed for each robot, and the final action of each robot is derived by weighting

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the relative importance of each behavior. In the leader-follower approach, a robot is designated as the leader, with the rest being followers. The followers robots need to position themselves relative to the leader and maintain a desired relative position with respect to the leader.

The strategy used in this work is the leader-follower approach. The system is a nonlinear dynamical system [11] and there are several methods to control the system presented in literature like backstepping [4], direct lyapunov method [12], feedback linearization [7], variable structure [8], sliding mode [6], neural network [5] and Fuzzy [14]. In this work, the method to realize the leader-follower formation control is the SDRE (State-Dependent Riccati Equation).

2 SYSTEM MODELING

The configuration of the system is showed in Figure 1 [12]. X-Y is the ground coordinates and x-y is the Cartesian coordinates fixed of the leader robot. (X_L,Y_L) and (X_F,Y_F) are global positions of the leader and follower respectively in which the subscripts 'L' and 'F' represent leader and follower respectively. v_L and v_F are leader's and follower's linear velocities, θ_L and θ_F are their orientation angles; ω_L e ω_F are leader's and follower's angular velocities. And l and φ are follower's relative distance and angle with respect to the leader.

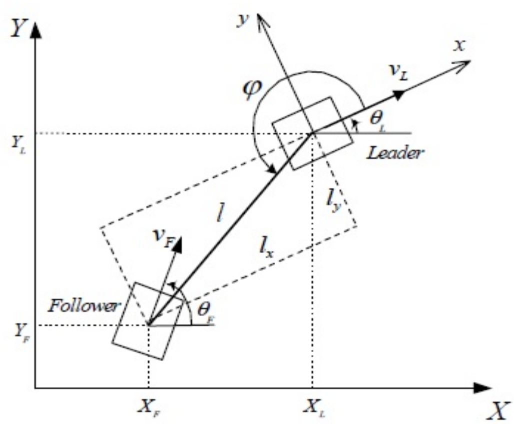


Figure 1: Configuration of the system [12].

The modeling of the nonlinear dynamical system is [12]:

$$\dot{e}_x = \omega_L e_y - v_F \cos e_\theta + f_1 \tag{2.1}$$

$$\dot{e}_y = -\omega_L e_x - v_F \sin e_\theta + f_2 \tag{2.2}$$

$$\dot{e}_\theta = \omega_F - \omega_L \tag{2.3}$$

$$f_1 = -l_d \dot{\varphi}_d \sin \varphi_d - \omega_L l_d \sin \varphi_d + v_L \tag{2.4}$$

$$f_2 = l_d \dot{\varphi}_d \cos \varphi_d + \omega_L l_d \cos \varphi_d \tag{2.5}$$

where $e_x = l_{xd} - l_x$, $e_y = l_{yd} - l_y$ and $e_\theta = \theta_F - \theta_L$.

Given v_L, ω_L, l_d and φ_d (d means desired), we need to find the control inputs v_F and ω_F in order to make $l_x \to l_{xd}$, $l_y \to l_{yd}$ and e_θ stable.

3 SDRE CONTROL METHOD

SDRE (State-Dependent Riccati Equation) control method have drawn an extensive research attention in control community recently [1]. This strategy is very efficient for nonlinear feedback controllers. The method represents the nonlinear system in a linear structure that have state-dependent matrices and minimizes a quadratic performance index. The algorithm solves, for each point in the state space, a algebraic Riccati equation and state-dependent. Because of this the method calls State-Dependent Riccati Equation.

Given the nonlinear system (2.1) to (2.5):

$$\dot{X} = f(X) + g(X)U \tag{3.1}$$

The system needs to be transformed in the following form:

$$\dot{X} = A(X)X + B(X)U \tag{3.2}$$

This process is called extended linearization. Extended linearization is the process of factorizing a nonlinear system into a linear-like structure which contains SDC (State Dependent Coefficient) matrices [1]. This linearization is non-unique because exist a lot of choices for $A(X)$ and $B(X)$.

The feedback control law that minimizes the quadratic performance index [10]:

$$J = \int_0^\infty \left[X(t)^T Q(X)X(t) + U(t)^T R(X)U(t)\right] dt \tag{3.3}$$

is:

$$U = -R^{-1}(X)B^T(X)P(X)X \tag{3.4}$$

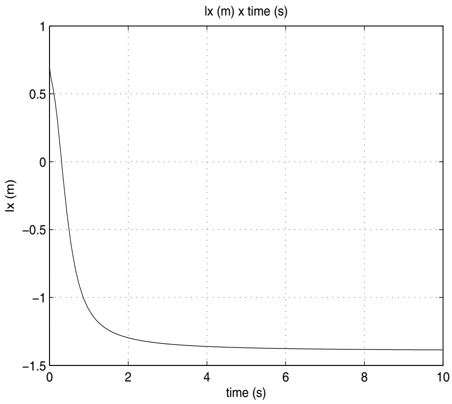
The matrix P(X) can be obtained by the algebraic Riccati equation:

$$P(X)A(X) + A^T(X)P(X) + Q(X) - P(X)B(X)R^{-1}(X)B^T(X)P(X) = 0 \tag{3.5}$$

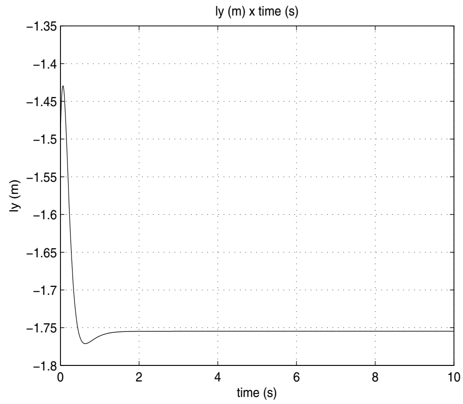
$Q(X)$ and $R(X)$ are weighting matrices and positive definite.

This method requires that the pair $(A(X), B(X))$ be controllable. This can be checked by forming the controllability matrix as in the linear systems sense and making sure that it has full rank in the domain of interest. This condition simply ensures that the algebraic Riccati equation has a solution at a particular state X. Due to the non-uniqueness of $A(X)$ and $B(X)$, different $A(X)$ and $B(X)$ choices may yield different controllability matrices and thus different SDRE controllability characteristics [1].

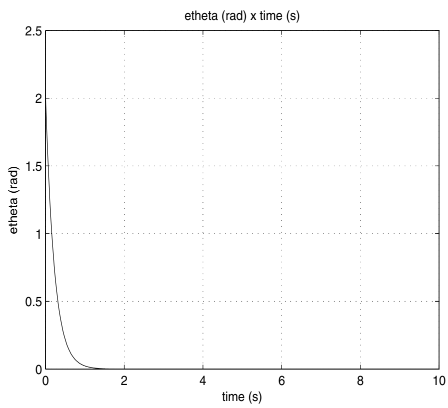
The SDRE control is not necessarily optimal, but suboptimal. This is also due to the non-uniqueness of $A(x)$ and $B(X)$ choices.



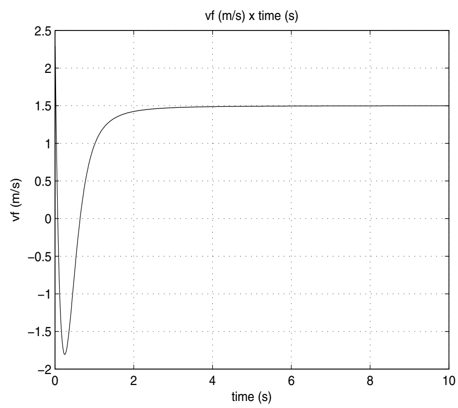
(a) Simulation 1 $- l_x$(m) $\times$ time(s)



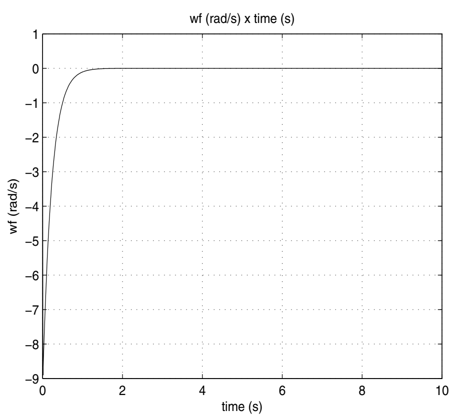
(b) Simulation 1 $- l_y$(m) $\times$ time(s)



(c) Simulation 1 $- e_\theta$(rad) $\times$ time(s)

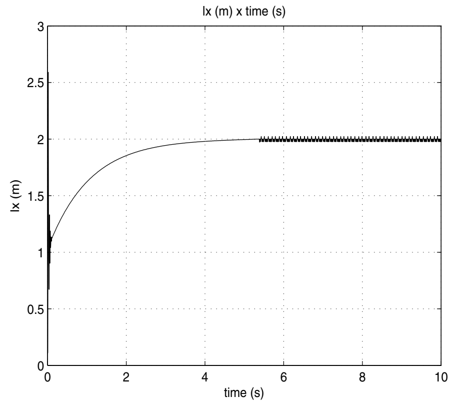


(d) Simulation 1 $- v_F$(m/s) $\times$ time(s)

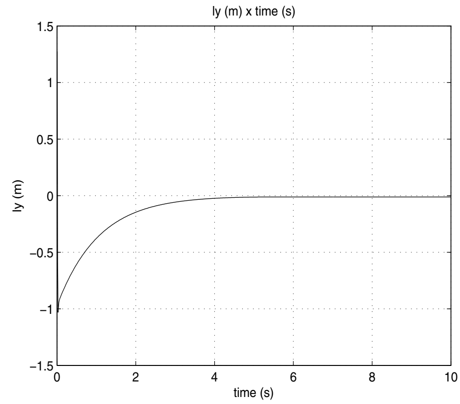


(e) Simulation 1 $- \omega_F$(rad/s) $\times$ time(s)

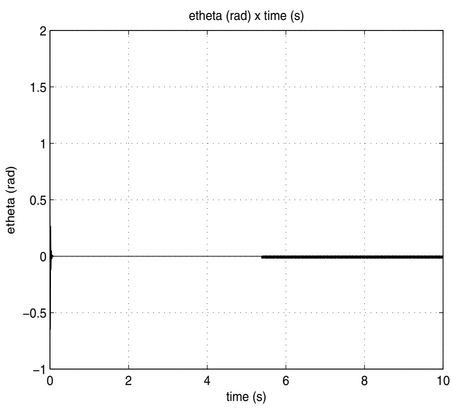
Figure 2: The leader moves along a straight line, and the follower keeps a constant relative distance and angle with respect to the leader.



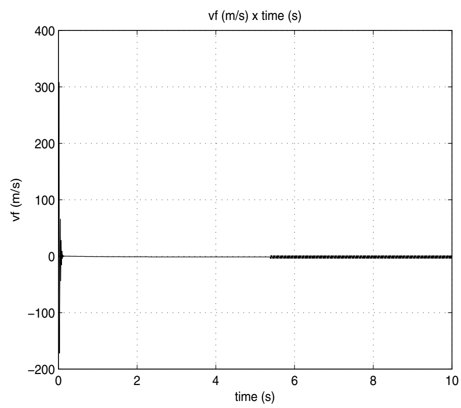
(a) Simulation 2 $- l_x(\text{m}) \times \text{time(s)}$



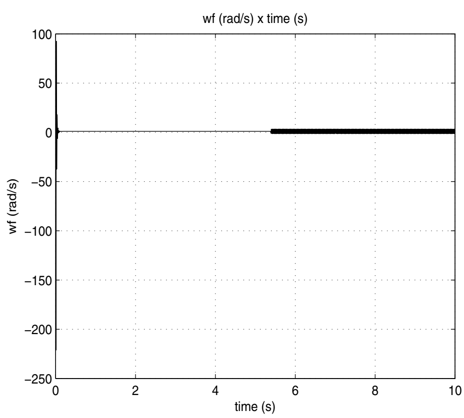
(b) Simulation 2 $- l_y(\text{m}) \times \text{time(s)}$



(c) Simulation 2 $- e_\theta(\text{rad}) \times \text{time(s)}$

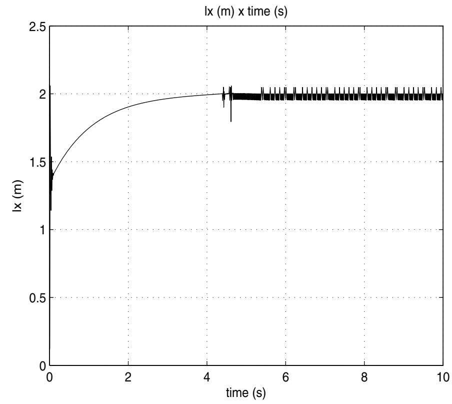


(d) Simulation 2 $- v_F(\text{m/s}) \times \text{time(s)}$

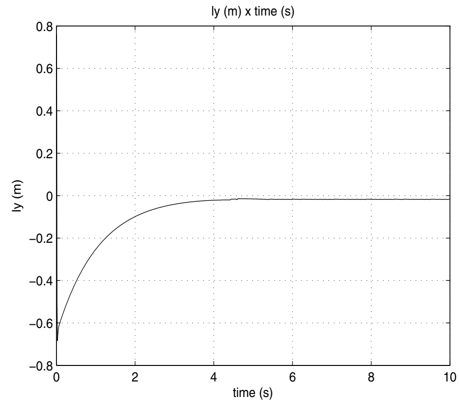


(e) Simulation 2 $- \omega_F(\text{rad/s}) \times \text{time(s)}$

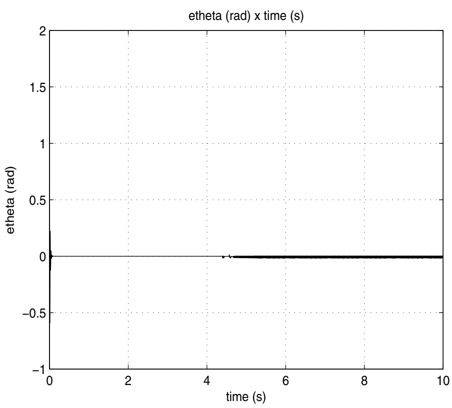
Figure 3: The leader moves goes along a circle, and the follower keeps a constant relative angle and distance with respective to the leader.



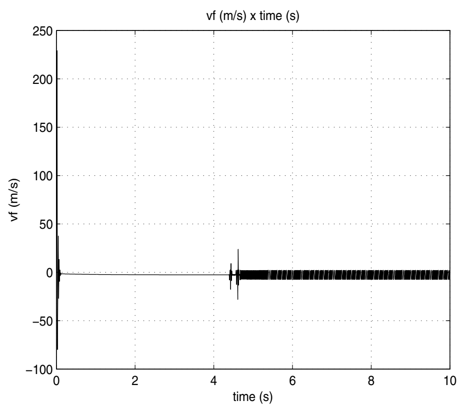
(a) Simulation 3 $- l_x(\mathrm{m}) \times \mathrm{time(s)}$



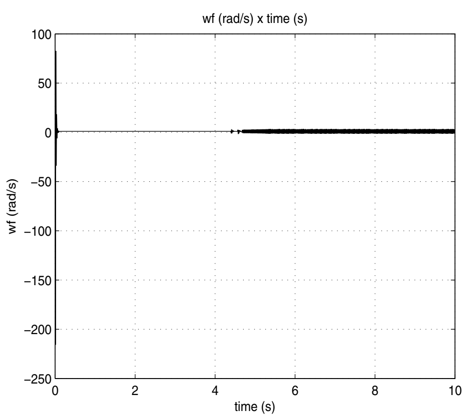
(b) Simulation 3 $- l_y(\mathrm{m}) \times \mathrm{time(s)}$



(c) Simulation 3 $- e_\theta(\mathrm{rad}) \times \mathrm{time(s)}$



(d) Simulation 3 $- v_F(\mathrm{m/s}) \times \mathrm{time(s)}$



(e) Simulation 3 $- \omega_F(\mathrm{rad/s}) \times \mathrm{time(s)}$

Figure 4: The leader moves goes along a circle, and the follower keeps a constant relative distance and rotates around the leader at a constant relative angular speed.

After construction of SDC form, the SDC matrices $A(X)$ and $B(X)$ are updated by states feedback periodically in each process time.

With the parameters $A(X)$, $B(X)$, $Q(X)$ and $R(X)$ given initially, the Riccati equation (3.5) need to be solved for $P(X)$. If exist a positive definite matrix $P(X)$, the system will be stable. With $P(X)$ the feedback control law (3.4) can be obtained. In this work the Ricatti equation was solved numerically with the software Matlab.

4 SIMULATION RESULTS

To analyze the performance of the controller we simulate three cases with the software Matlab. In the first case $\omega_L = 0$, i.e., the leader's heading direction does not change. The leader moves in a constant linear speed of $v_L = 1.5$ m/s along a straight line with $\theta_L = \pi/6$ mad and the follower keeps a constant relative distance $l_d = 2.0$ m and a constant relative angle $\varphi_d = 1.25\pi$ rad from the leader ($l_{xd} = l_{yd} = -1.41$ m). The initial conditions are $l_{x0} = 0.7$ m, $l_{y0} = -1.5$ m and $e_\theta = 0.65\pi$ rad. In the second case $\omega_L = 0.3\pi$ rad/s and $v_L = 0.5$ m/s. The follower keeps a constant relative distance $l_d = 2.0$ m and a constant relative angle $\varphi_d = 0.5\pi$ rad from the leader ($l_{xd} = 2.0$ m $l_{yd} = 0$ m). The initial conditions are $l_{x0} = 0.1$ m, $l_{y0} = 0.1$ m and $e_\theta = 0.5\pi$ rad. The third case is equal to the second, the only difference is that the follower rotates around the leader at a constant relative angular speed of $\dot{\varphi} = 0.2\pi$ rad/s. The numeric method to solve the nonlinear system is the Euler method [2].

Analyzing the results of the simulations we can see that the proposed controller can achieve the desired formation, and the whole system is stable.

5 CONCLUSIONS AND FUTURE WORKS

In this work we presented a study concerning the modeling and formation control of a robotic system composed by two mobile robots, the leader robot and the follower robot. It was applied the SDRE method in the leader-follower formation control strategy. Analyzing the results we can see that the system is stable and have a good performance.

The main future works that could be realized is modeling the system with more than two robots, try another kind of control method and considering problems like obstacle avoidance in the environment and path planning.

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RESUMO. Controle de formação de múltiplos robôs móveis é uma área de pesquisa relativamente nova da robótica e melhora o desempenho do controle e vantagens de sistemas de múltiplos robôs. Neste trabalho apresentamos um estudo relacionado com a modelagem e controle de formação de sistemas robóticos compostos por dois robôs móveis, o robô líder e o robô seguidor. Trata-se de um sistema dinâmico não linear e não pode ser controlado por técnicas de controle tradicionais. A estratégia de controle proposta é o método SDRE (Equação de Ricatti Dependente do Estado). Resultados simulados com o software Matlab mostram a eficiência do método de controle.

Palavras-chave: controle de formação, sistemas multi-robôs, robôs móveis, sistemas dinâmicos não lineares, controle SDRE.

REFERENCES

[1] T. Çimen. State-Dependent Riccati Equation (SDRE) Control: a Survey, in “Proceedings of the 17th World Congress the International Federation of Automatic Control”, Seoul, Korea (2008).

[2] S.C. Chapra & R.P. Canale. “Numerical Methods for Engineers”. The McGraw-Hill Companies, (2001).

[3] J. Desai, J. Ostrowski & R. Kumar. Modeling and Control of Formations of Nonholonomic Mobile Robots. *IEEE Transactions on Robotics and Automation*, **17** (2001), 905–908.

[4] T. Dierks & S. Jagannathan. Control of Nonholonomic Mobile Robot Formations: Backstepping Kinematics into Dynamics, in “Proceedings of the International Conference on Control Applications Part of IEEE Multi-Conference on Systems and Control”, Singapore (2007).

[5] T. Dierks & S. Jagannathan. Neural Network Output Feedback Control of Robot Formations. *IEEE Transactions on Systems, Man, and Cybernetics*, **40** (2010).

[6] S. Dongbin, S. Zhendong & Q. Yupeng. Second-Order Sliding Mode Control for Nonholonomic Mobile Robots Formation, in “Proceedings of the 30th Chinese Control Conference”, Yantai, China (2011).

[7] S.S. Ge & F.L. Lewis. “Autonomous Mobile Robots – Sensing, Control, Decision Making and Applications”, Taylor and Francis Group, (2006).

[8] Q.P. Ha, H.M. Ha & G. Dissnayake. Robotic Formation Control Using Variable Structure System Approach, in “Proceedings of the IEEE Workshop on Distributed Intelligent Systems: Collective Intelligence and Its Applications”, Sydney, Australia (2007).

[9] J.S. Jennings, G. Whelan & W.F. Evans. Cooperative Search and Rescue with a Team of Mobile Robots, in “Proceedings of the International Conference on Advanced Robotics”, Monterey, USA (1997).

[10] D.E. Kirk. “Optimal Control Theory – An Introduction”. Dover Publications, (1970).

[11] H.K. Khalil. “Nonlinear Systems”, Prentice Hall, (2002).

[12] X. Li & J. Xiao. Formation Control in Leader-Follower Motion Using Direct Lyapunov Method. *International Journal of Intelligent Control and Systems*, **10** (2005), 244–250.

[13] Z. Wang, Y. Mao, G. Chen & Q. Chen. Leader-Follower and Communications Based Formation Control of Multi-Robots, in “Proceedings of the 10th World Congress on Intelligent Control and Automation”, Beijing, China (2012).

[14] E. Yang & D. Gu. A Multi-agent Fuzzy Policy Reinforcement Learning Algorithm with Application to Leader-Follower Robotic Systems, in “Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems”, Beijing, China (2006).