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Multiscale analysis of GPS time series from non-decimated wavelet to investigate the effects of ionospheric scintillation [1]

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Abstract. Due to the numerous application possibilities, the theory of wavelets has been applied in several areas of research. The Discrete Wavelet Transform is the most known version. However, the downsampling required for its calculation makes it sensitive to the origin, what is not ideal for some applications, mainly in time series. On the other hand, the Non-Decimated Discrete Wavelet Transform (or Maximum Overlap Discrete Wavelet Transform, Stationary Wavelet Transform, Shift-invariant Discrete Wavelet Transform, Redundant Discrete Wavelet Transform) is shift invariant, because it considers all the elements of the sample, by eliminating the downsampling and, consequently, represents a time series with the same number of coefficients at each scale. In the present paper, the objective is to present the theorical aspects of the a multiscale/multiresolution analysis of non-stationary time series from non-decimated wavelets in terms of its implementation using the same pyramidal algorithm of the decimated wavelet transform. An application with real time series of the effect of the ionospheric scintillation on artificial satellite signals is investigated. With this analysis some information and hidden patterns which can not be detected in the time domain, may therefore be explained in the space-frequency domain.

Keywords. Non-decimated wavelets, maximum overlap discrete wavelet transform, multiescale analysis, time series, ionospheric scintillation, GPS.

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1. Introduction

The theory of wavelets has been quite widespread, providing major advances in several different areas of the science. There are numerous possibilities of aplication, being used for modeling, decomposition, removal of noise and/or unwanted effects, compression, detection of singularities, among other applications.

In general, the decimated version (Discrete Wavelet Transform - DWT) is the most known. It is calculated through an efficient algorithm developed by Mallat, called Pyramid Algorithm, which uses discrete filters and downsampling. This process eliminates the even or odd samples, so that at level j there is half of the coefficients of the previous level $j-1$. Applying an odd downsampling is similar to applying an even downsampling on data shifted by one unit. The DWT depends on the choice of dowmsampling, which corresponds to the choice of an origin.

In this application, the multiscale analysis is applied to non-stationary time series (TS) from artificial satellites signals. Therefore, it is necessary that the method is not sensitive to the origin, i.e., it is preferable that it is shift invariant, that is obtained with Non-Decimated Wavelet Transform (NDWT) [5] or Maximal Overlap Discrete Wavelet Transform (MOWT) [7]. The MOWT takes into account all the elements: even and odd, making a signal be represented with the same number of coefficients at each scale.

One of the motivations for the development and use of the NDWT theory that will be presented, is the implementation. Its is possible implement the NDWT applying the DWT pyramid algorithm is applied twice, first in the original vector and then in the shift vector, eliminating the downsampling and causing that all the elements are considered at each stage of the algorithm.

The growing influence of GPS (Global Positioning System) in the navigation and remote sensing, is quite evident nowadays. However, until satellite signal gets to the GPS receiver, effects, often severe, influence the signal, causing errors or even the loss of the satellite signal by the receiver.

When the signal passes by the ionosphere, which is a plasmatic region of the atmosphere that contains ions and electrons in sufficient quantity to affect any signal that passes through it. The ionospheric scintillations, which are irregularities in the density of ions in the ionosphere is even worse. Its effects are more severe at high latitudes, where their occurrence is related to periods of high solar activity, magnetic storms and other extreme activities, and in equatorial regions and regions of low latitudes, where the equatorial ionization anomaly (EIA) occurs, mainly in the interval after sunset and before midnight. Brazil is "privileged" to study the ionosphere because its location is in the equatorial zone suffering the intense effects of the ionospheric scintilation and the impact of high solar activity.

For high-precision applications it is necessary the correct understanding of the effect of the ionospheric scintillation on GPS signals, enabling the characterization of this effect and thus reducing their interference.

In this paper it is intended to investigate the behavior of the effect of the ionospheric scintillation on GPS signals from a TS approach that goes beyond of the classical methods, which assumes stationarity [4]. Due to the irregularities and scattering of the F layer of the ionosphere, the effects of scintillation in the satellite signals may not be stationary. The use of the NDWT for a complete analysis in multiscale will make information and hidden patterns that could not be detected in the time domain can be explained in the space/frequency domain.

2. Theoretical Aspects

Wavelet is the function $\psi(t) \in L^2(\mathbb{R})$ that satisfies the following properties [7]:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \text{ e } \int_{-\infty}^{\infty} \psi^2(t)dt = 1.$$

There are two known functions that perform a primary role in the Wavelet Analysis, the scaling function ϕ, called father *wavelet* and the *wavelet* ψ, also called mother *wavelet*.

The father *wavelet*, can be expressed as

$$\phi(t) = \sqrt{2}\sum_l h_l \phi(2t - l)$$

where h_l is a low-pass filter, called *wavelet* filter.

On the other hand, the relationship between the filters h_l and g_l,

$$g_l = (-1)^l h_{1-l}$$

called quadrature mirror filter relation, it is considerated that the mother wavelet ψ can be defined as

$$\psi(t) = \sqrt{2}\sum_l g_l \phi(2t - l),$$

where g_l is a high-pass filter called scaling filter.

Through dilations and translations appropriate for ϕ and ψ, described by

$$\phi_{j,l}(t) = 2^{-j/2}\phi(2^{-j}t - l) \text{ and } \psi_{j,l}(t) = 2^{-j/2}\phi(2^{-j}t - l),\ j, l \in \mathbb{Z},$$

it is possible to construct bases for various function spaces. This is the main motivation for the development of the Wavelet Transform (WT), whose goal is to expand $f \in L^2(\mathbb{R})$ using a set of wavelet functions that have space/frequency location.

This way, it is considered that any signal $f(t) \in L^2(\mathbb{R})$ can be represented as [3]

$$f(t) = \sum_l c_{j_0,l}\phi_{j_0,l}(t) + \sum_{j \geq j_0} \sum_l d_{j,l}\psi_{j,l}(t),$$

where

$$\begin{aligned} c_{j_0,l} &= \langle f(t), \phi_{j_0,l}(t) \rangle = \textstyle\int_{-\infty}^{\infty} f(t)\phi_{j_0,l}(t)dt, \\ d_{j,l} &= \langle f(t), \psi_{j,l}(t) \rangle = \textstyle\int_{-\infty}^{\infty} f(t)\psi_{j,l}(t)dt, \end{aligned} \tag{2.1}$$

and j_0 represents the lowest level of resolution.

The coefficients described in (2.1), called smooth (or scaling) and detail (or wavelet) coefficients, respectively, are result of the filtering of the function $f(t) \in L^2(\mathbb{R})$, by wavelets and scaling filters.

Aiming at practical application with sampled signals the following notation of DWT, in particular the NDWT, will be in the sense to facilitate its implementation.

The great motivation for the use of the NDWT in this work is to use a transform that operates similarly to DWT, but does not suffer sensitivity to the origin choice for a TS, i.e., it is shift invariant. This sensitivity of the DWT is due entirely to dowsample the output from the wavelet and scaling filters at each scale (or level) of the pyramid algorithm, where in each two outputs of the filter, one output is discarded.

The idea of the NDWT is to eliminate the downsampling, considering all the elements, and representing a TS with the same number of coefficients at each scale.

The outputs of the filters that are discarded at the first level of the DWT pyramid algorithm, can be obtained by applying the DWT pyramid algorithm in the shift vector TX instead of X, where X is a sequence of observations (or TS). This suggests that the NDWT can be obtained by applying the usual DWT pyramid algorithm twice, once to X and the other to TX, and after merging the two sets of DWT coefficients together [7].

The NDWT wavelet $\tilde{W}_j$ and scaling $\tilde{V}_j$ coefficients, of level j, are calculed using the scaling coefficients $\tilde{V}_{j-1}$ of level $j-1$, as in DWT. However, downsampling does not occur and the filters are modified at each scale.

The NDWT wavelet and scaling coeffients can be seen as the result of the filtering of a TS X with the NDWT wavelet and scaling filters, which are presented below.

The NDWT wavelet $\left\{\tilde{h}_l\right\}$ and scaling $\{\tilde{g}_l\}$ filters, must satisfy

$$\sum_{l=0}^{L-1} \tilde{h}_l = 0, \ \sum_{l=0}^{L-1} \tilde{h}_l^2 = 1/2 \ \text{ and } \ \sum_{-\infty}^{+\infty} \tilde{h}_l \tilde{h}_{l+2n} = 0,$$

and

$$\sum_{l=0}^{L-1} \tilde{g}_l = 1, \ \sum_{l=0}^{L-1} \tilde{g}_l^2 = 1/2 \ \text{ and } \ \sum_{-\infty}^{+\infty} \tilde{g}_l \tilde{g}_{l+2n} = 0,$$

respectively, for a nonzero integer n.

So, given a TS $\{X_t : t = 0, ..., N-1\}$, the result of the filtering of $\{X_t\}$ with the NDWT wavelet and scaling filters is given, respectively, by

$$\begin{array}{c} \tilde{W}_{j,t} = \sum_{l=0}^{L-1} \tilde{h}_{j,l} X_{t-lmodN} \\ \tilde{V}_{j,t} = \sum_{l=0}^{L-1} \tilde{g}_{j,l} X_{t-lmodN}, t = 0, 1, ..., N-1. \end{array} \tag{2.2}$$

These two sequences given by equation (2.2) represent the NDWT of level j, and stipulate that the elements of $\tilde{W}_j$, $\tilde{V}_j$ and $\tilde{V}_{j-1}$ are obtained by circularly filtering $\{X_t\}$ with the filters $\left\{\tilde{h}_{j,l}\right\}$, $\{\tilde{g}_{j,l}\}$ and $\{\tilde{g}_{j-1,l}\}$, respectively. The term *mod* of this equation allows a circularly filtering making X to be represented with the same numbers of coefficients at each scale.

Moreover, it can be shown (see section 5.5 [7]) that it is possible to obtain $\tilde{W}_j$ and $\tilde{V}_j$ by filtering of $\tilde{V}_{j-1}$ by the following equation

$$\begin{array}{c} W_{j,t} = \sum_{k=0}^{K-1} \tilde{h}_k \tilde{V}_{j-1,t-2^{j-1}l mod \text{ N}} \\ \tilde{V}_{j,t} = \sum_{k=0}^{K-1} \tilde{g}_k \tilde{V}_{j-1,t-2^{j-1}l mod \text{ N}},\ t = 0, 1, ..., N-1 \end{array} \tag{2.3}$$

The equations in (2.3) represent the NDWT pyramid algorithm and they can be written in matrix form as

$$\tilde{W}_j = \tilde{B}_j \tilde{V}_{j-1} \text{ and } \tilde{V}_j = \tilde{A}_j \tilde{V}_{j-1},$$

where the rows of $\tilde{B}_j$ contain circularly shifted versions of $\left\{\tilde{h}_j\right\}$ after it has been upsample to width $2^{j-1}(L-1)+1$ (that consists of insert 2^{j-1} zeros between each of the L values of the original filter) and then periodized to lenght N, and with a similar construction for $\tilde{A}_j$ based upon $\{\tilde{g}_j\}$ [7].

The NDWT also allows to reconstruct $\tilde{V}_{j-1}$ of $\tilde{W}_j$ and $\tilde{V}_j$. The inverse DWT can be calculated via inverse pyramid algorithm described by the following equation

$$\tilde{V}_{j-1,t} = \sum_{l=0}^{L-1} \tilde{h}_l \tilde{W}_{j,t+2^{j-1}l mod N} + \sum_{l=0}^{L-1} \tilde{g}_l \tilde{V}_{j,t+2^{j-1}l mod N},\ t = 0, 1, ..., N-1,$$

or, in matrix form,

$$\tilde{V}_{j-1} = \tilde{B}_j^T \tilde{W}_j + \tilde{A}_j^T \tilde{V}_j.$$

Through the wavelet periodogram, the energy of the process (or TS) $\{X_t\}$ is decomposed in different scales and with space/frequency location. It is calculated from the NDWT wavelet coefficients given in (2.2) and can be described by [3]

$$I_{j,t} = \left|\tilde{W}_{j,t}\right|^2 \tag{2.4}$$

where $\tilde{W}_{j,t} = \sum_{l=0}^{L-1} \tilde{h}_{j,l} X_{t-l mod N}$, $t = 0, 1, ..., N-1$.

Each coefficient of (2.4) is represented through a vertical line drawn from a horizontal reference line for each level. The coefficients at each level are spaced so that they represent their location.

The periodogram provides a good description of where the significant changes are located in the function. Abrupt changes in the function correspond proportionately large magnitude of the coefficients.

From of the wavelet periodogram, a decomposition of variances is gotten in multiscale of the TS, i.e., in different levels of resolution j. Based on these results, it is possible to calculate the variance (energy) at each scale, with the goal to know what scale has more energy. This task can be executed by summing all coefficients at each scale, according to the following equation

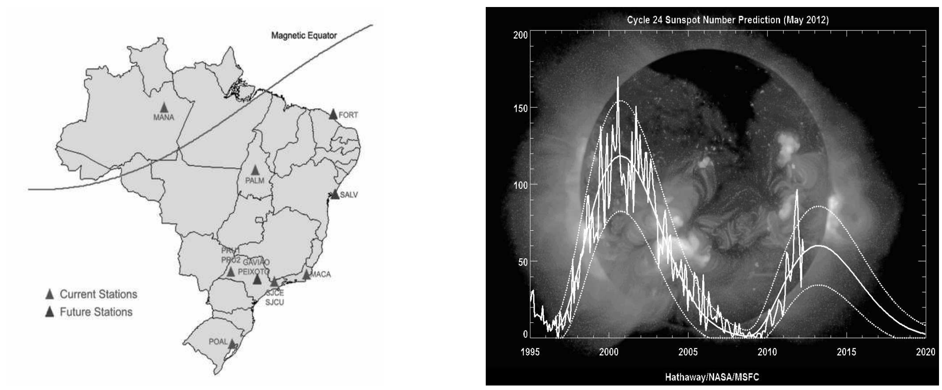
$$S_{j,t;n} = \frac{1}{N} \sum_{n=0}^{N-1} \left| \tilde{W}_{j,t;n} \right|^2 . \tag{2.5}$$

and we have the global wavelet spectrum.

3. Application and Results

The importance of the study of the ionospheric scintillation in the equatorial region is so significant that in March 2010, the CIGALA (Concept for Ionospheric Scintillation Mitigation for Professional GNSS in Latin America) project was started, with the objective of studying the causes of the ionospheric scintillation on low latitudes, as well as its effects on technologies under development. The choice of Latin America, especially Brazil is due to two reasons: it is in one of the most critical regions under effect of ionospheric irregularities and it has academic and commercial institutions of great relevance to implementation and development of the project (UNESP, INPE, Petrobras).

With this project, 6 stations with specific high frequency (50 Hz) receptors (Septentrio PolaRxS) were installed in strategic places, as illustrated in Figure 1(a), so that the collections, maintenance and re-occupations are possible, for validation of possible methodologies and algorithms developed. Thus, extreme important data for the study of the ionospheric scintillation are available since February 14, 2011.



(a) (b)

Figure 1: (a)Stations CIGALA GPS to collect data to calculate the scintillation indexes in Brazil; (b)Cycle 24 - period of maximum solar activity in 2013.

In order to investigate the TS of the effect of the ionospheric scintillation on artificial satellite signals, and evaluate the methodology proposed, the data of the

CIGALA Project available for the entire year of 2012 were used. The PRU1 station located at UNESP, Presidente Prudente, was chosen and the GPS satellite 11 was selected due to its highest scintillation index. The receiver, a Septentrio PolaRxS, represents the state of the art on receivers for tracking multi constellations and triple-frequency in the monitoring of the ionosphere. It has OXCO technology for extremely low noise and allows gathering intervals up to 50 hz. It is possible to obtain the $S4$ and $\sigma\phi$ scintillation indexes that can be used to characterize and help to understand the ionospheric irregularities that cause scintillation, as well as establish strategies for forecasting effects of the scintillation that can make the GPS ineffective [8].

The amplitude index $S4$ is the indicator of ionospheric scintillation. If $S4 \geq 1.0$ the level of scintillation is classified as strong, when $0.5 \leq S4 \leq 1.0$ the scintillation is moderate, and for $0 \leq S4 \leq 0.5$ the scintillation is weak.

The ionosphere is a very unstable layer that undergoes variations on several time scales - during the day, throughout the seasons and solar cycles that occur at each 11 years (Figure 1(b)). In 2012, the highest index $S4$ occured in March, as shown in Figure 2, where the index $S4$ exceeds 2.1.

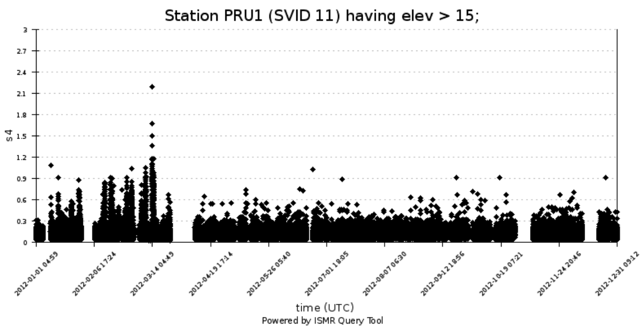


Figure 2: Scintillation index $S4$ from 01/01/2012 to 12/31/2012.

With a "zoom" on period from 03/12/2012 to 03/14/2012, it can be seen more clearly in Figure 3 that the TS of scintillation index presents gaps caused by the lack of data when the satellites are not being tracked, and periodic behavior at each day where there are data.

This periodic behavior may be related to multipath effects due to reflection of satellite signals on surfaces nearby of the receptor. If the receiver environment remains unchanged, the multipath effect changes according to the movement of the satellite, which has daily repeatability. It is noted also that this periodic behavior has a shape of "U". Whereas the elevation angle of the satellite follows the behavior of a parabola with concavity facing downwards, it is expected that the reflections (multipath) and noises are more expressive to lower angles, as shown in Figure 4.

It is important to eliminate or separate such behavior that not characterizes scintillation because it may influence the analysis of ionospheric scintillation index $S4$.

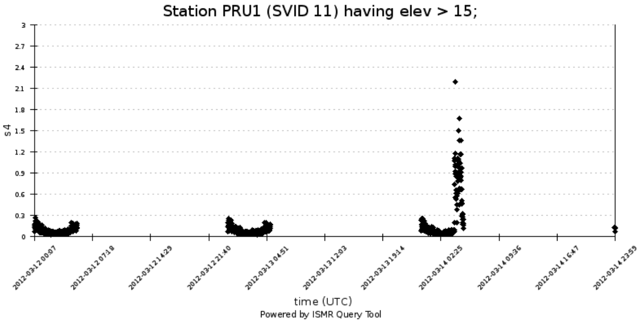


Figure 3: Scintillation index $S4$ from 03/12/2012 to 03/14/2012 (period of strong scintillation).

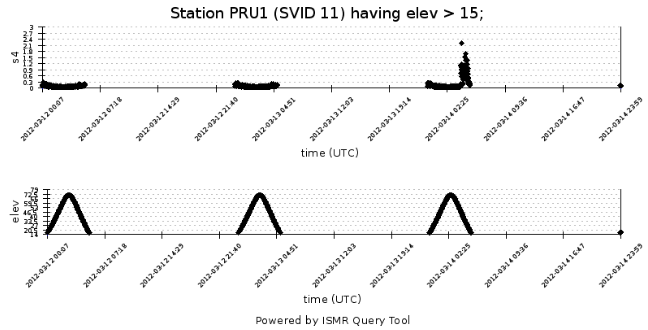
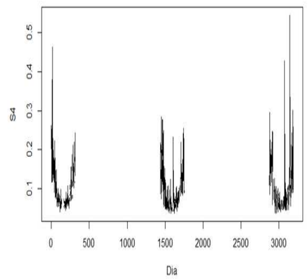


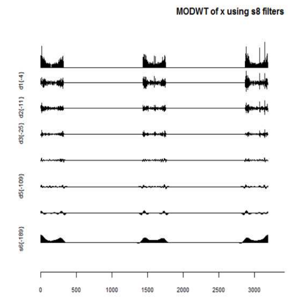
Figure 4: Scintillation index $S4$ from 03/12/2012 to 03/14/2012 with elevation angle of the satellite 11 plotted in the graph below.

The MOWT was applied to obtain the wavelet and scaling coefficients, given by 2.2, considering the wavelet family Symmlets with 10 vanishing moments (SYM10)[2].

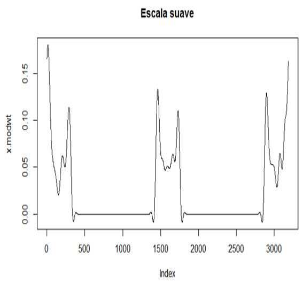
In the Figure 5, has on (a) the TS of the index $S4$ that caracterize weak ionospheric scintillation, for comparision. Extracting information in multiscale, as theory presented in section 2, we obtain in (b) the daily behavior of the TS in Figure 3 evidenced at their 3 smothest scales. So, we propose in this paper estimate this behavior, which is evidenced at smoother scales of the wavelet periodogram and subtract it from the TS. The estimation will be performed by multiscale decomposition of the period from 06/12/2012 to 06/14/2012 with low scintillation index, as preconized by the literature. Thus, the 3 smothest scales are reconstructed in (c) to obtain a estimation of such behavior (or effect). For a more detailed analysis in multiscale, was plotted (Figure 5(d)) the wavelet spectrum estimated by the wavelet periodogram, described by the equation 2.4, and the global spectrum, which represents the energy or the total variance at each level of resolution or scale that refers to this level. We can note in the global wavelet spectrum, equação 2.5, that the scale of level of resolution 9 is more relevant. As the 9^{th} level is related to the effects of 512 to 1024 minutes, so this demonstrates that this effect is the closest of the daily behavior, which was expected and reinforces the existence of the multipath on scintilattion index $S4$.



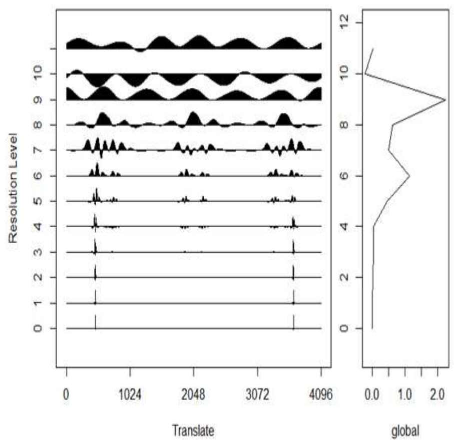
(a) Scintillation index $S4$ from 06/12/2012 to 06/14/2012 (period of weak scintillation).



(b) Multiscale decomposition



(c) Smoother scales reconstructed



(d) Periodogram of the smoother scales

Figure 5: Estimation of the periodic behavior presented in the ST of scintillation index $S4$.

Having extracted the daily behavior of the TS of index $S4$ of the period of strong scintillation (03/12/2012 to 03/14/2012), a TS free of any interference of cyclical effect is plotted in Figure 6.

Decomposing in multiscale the TS plotted in Figure 6, we can observed in Figure 7 that the scale relative to level of resolution 6 of the periodogram is most evident, i.e., it concentrates higher energy. This scale is closest to the effects of 1 hour. Having removed the multipath effect of the serie, the periodic effects is not expressive in the spectrum anymore, not influencing the analysis of the scintillation index TS.

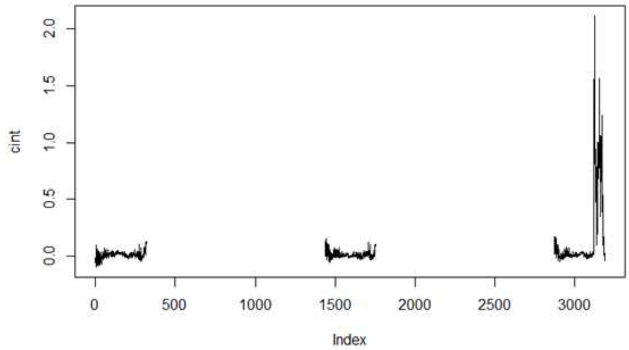


Figure 6: Index $S4$ of the period with strong scintillation, with smoother scales (estimated in the period without scintillation) removed.

It is also important to notice that the peak of the scintillation was very evident in the wavelet analysis, showing the behavior of the most abrupt change in the series of index $S4$ and where it occurs.

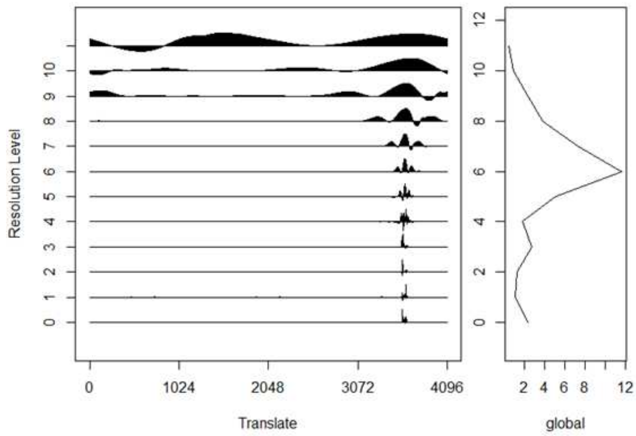


Figure 7: Periodogram of the scintillation index with smoother scales removed.

4. Conclusions

In this article a new methodology is presented for the investigation of the ionospheric scintillation through the wavelet spectrum of the TS of the index $S4$ on GPS signals.

It was possible to give a first step in relation to separate scintillation effect from other effects (mainly multipath) that may influence the $S4$ index. After the estimation and removal of the periodic effect (multipath), a series free of any periodic effect was obtained, making possible the investigation of ionospheric scintillation.

It is noteworthy that for effective analysis of the scintillation it is necessary to analyze a longer period of data, all available satellites and the other stations.

Resumo. Devido às inúmeras possibilidades de aplicação, a teoria de *wavelets* tem sido utilizada nas mais diversas áreas de pesquisa. A Transformada Wavelet Discreta é a versão mais conhecida. Porém, o processo de decimação necessário para seu cálculo, faz com que ela seja sensível à origem, o que para algumas aplicações não é o ideal. Ao contrário, a Transformada Wavelet Discreta Não Decimada é invariante à translação, pois leva em consideração todos os elementos da amostra, de modo a representar uma série temporal com o mesmo número de coeficientes em cada escala, tendo o processo de decimação eliminado. Neste artigo pretende-se realizar uma análise multiescala/multirresolução de séries temporais não estacionárias a partir de *wavelets* não decimadas. Tal procedimento será aplicado na investigação do efeito da cintilação ionosférica nos sinais de satélite artificiais, de modo que informações e padrões escondidos, que não podem ser detectados no domínio do tempo, podem assim, ser explicitados no domínio espaço/freqüência.

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